

Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach*

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February 5, 2020

Abstract

This paper uses a risk-shifting model to analyze policy responses to asset price booms. We show risk shifting leads to inefficient asset and credit booms in which asset prices can exceed fundamentals. However, the inefficiencies associated with risk-shifting arise independently of whether the asset is a bubble. Given evidence of risk-shifting, then, policymakers may not need to determine if assets are bubbles to justify intervention. We then show that some of the main candidate interventions against asset booms have ambiguous welfare implications: Tighter monetary policy can exacerbate some inefficiencies but mitigates others, while leverage restrictions can raise asset prices and lead to more excessive leverage. Policy responses are more effective when they disproportionately discourage riskier investments.

*We thank Robert Barsky, Marco Bassetto, Huberto Ennis, Tomoyuki Nakajima, Toan Phan, Alp Simsek, and Sanjay Singh for comments, as well as audiences at various conferences and seminars. The views here do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

Introduction

Policymakers have long debated how to respond to asset booms and potential bubbles, i.e. situations where asset prices surge to levels that seemingly exceed the value of dividends these assets are expected to yield. One view, summarized in Bernanke and Gertler (1999) and Gilchrist and Leahy (2002), argues policymakers should wait to see what happens to asset prices and act only if asset prices collapse and drag down economic activity. An alternative view, summarized in Borio and Lowe (2002) and reinforced in subsequent work by Jorda, Schularick, and Taylor (2015) and Mian, Sufi, and Verner (2017), argues asset booms are likely to end in financial crises and recessions, especially when they coincide with credit booms. By intervening to dampen asset prices during booms, they reason, policymakers might mitigate the eventual crash.

The severity of the Global Financial Crisis in 2007 and the difficulty central banks faced in providing stimulus in its wake led many policymakers to lean toward a more proactive response to asset booms. This shifted the debate from *whether* to intervene to *how* to intervene. The leading proposals for intervention include monetary tightening and macroprudential regulation. Both approaches have at times been criticized. For example, Svensson (2017) argues the costs of monetary tightening during asset booms exceed its benefits. In the opposite direction, Stein (2013) argues that even if regulatory policy could work in principle, in practice it is likely to be circumvented through clever financial engineering.

This paper tackles the question of how policy should respond to asset booms through the lens of a risk-shifting model, meaning that the lenders who ultimately finance asset purchases cannot gauge the default risk they face from any individual borrower. We focus on risk-shifting because asset booms often feature extensive lending against assets that are hard for lenders to evaluate, either because they are tied to new and imperfectly understood technologies (dot-com, blockchain, tranching securities) or because they are valued idiosyncratically, like housing, making it hard to distinguish committed home buyers from speculators who will walk away if house prices fall.¹ To be sure, there is a vast literature on asset booms and bubbles that abstracts from risk shifting, so our mechanism is not essential for booms or bubbles. However, as we elaborate in the Conclusion, risk-shifting can naturally emerge in those models as well. Our analysis should thus be viewed as complementary to alternative models of bubbles rather than a substitute.

Our focus on risk-shifting mirrors recent work by Aikman, Haldane, and Nelson (2015) and Martinez-Meira and Repullo (2017) that uses risk-shifting to analyze credit booms and banking crises. In these papers, banks attract deposits and then invest them in investments of varying risk. These papers show how shocks to productivity or desired savings can lead banks to fund riskier investments that are more likely to end in default. Since these models do not feature assets, it is hard to relate them to asset booms or bubbles. They also abstract from how attributes of the underlying assets might matter for the boom.

¹While we describe situations where limited information is an exogenous feature of an asset, Asriyan, Laeven, and Martin (2018) argue asset booms can reduce the incentive to screen borrowers, so information about assets deteriorates endogenously.

Our model builds on existing work on risk-shifting and asset pricing. Allen and Gorton (1993) were the first to show that risk-shifting allows asset prices to exceed fundamentals, an idea further developed in Allen and Gale (2000), Barlevy (2014), Dow and Han (2015), Dubecq, Mojon, and Ragot (2015), and Bengui and Phan (2018). We contribute to this literature in two ways. First, we use a general equilibrium setup that can incorporate policy interventions absent in previous work. Second, we introduce costly default, allowing us to capture a fall in output when an asset boom ends. Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that asset price crashes are associated with a fall in GDP per capita of 9-16%; Atkinson, Luttrell, and Rosenblum (2013) estimate even larger cumulative losses for the US in the recent crisis. Economists have identified various reasons for why output falls when asset prices collapse. For example, financial intermediaries who lent against assets may not be able to finance new investments when they face an overhang of debt against those assets. Alternatively, indebted households may delever when the assets they borrowed to buy fall in price, and with nominal price rigidity such deleveraging can reduce aggregate demand and output.² Default costs similarly leave agents poorer when asset prices collapse, although this is because lenders must use resources to recover their obligations. Allowing for such a contraction, even in this stylized way, has important implications for policy.

At the heart of our model is an information asymmetry in which borrowers know the risks of their investments better than lenders. This encourages agents to borrow and gamble on risky assets, knowing it will be lenders who bear the losses and default costs if the gamble fails. As speculators buy up assets, they drive up asset prices and drive down the expected return on assets. This leads to two distinct inefficiencies. The first is misallocation. Borrowers direct too many resources to risky investments that offer high private returns but low overall returns. This is consistent with evidence of misallocation during credit booms in Borio, Kharroubi, Upper, and Zampolli (2015) and Charles, Hurst, and Notowidigdo (2018). The second inefficiency involves excessive leverage as agents ignore the default costs they impose on others and borrow more than is socially optimal. Both of these distortions arise because agents who borrow to buy risky assets fail to account for the externalities they impose on their lenders in the form of losses and default costs.

Our model gives rise to equilibria that are broadly consistent with historical asset booms: Asset prices appear excessive and can grow exponentially, the asset boom is accompanied by a credit boom, borrowing to buy risky assets is relatively cheap, and realized returns on assets are high. The boom can feature a bubble where asset prices exceed fundamentals. But, unlike in previous work, we find that the boom is inefficient even without giving rise to a bubble. The role for policy in our model is not to push prices toward fundamentals, but to correct distortions that arise in lending markets where those who borrow to speculate are subsidized by safer borrowers. In contrast to Bernanke and Gertler (2001) who argue policymakers should not intervene when they are unsure if they face a bubble or not, our model provides justification for the Borio and Lowe (2002) position that policymakers should intervene even without knowing if asset

²See Phillipon (2010) on debt overhang and Korinek and Simsek (2016) and Farhi and Werning (2016) on aggregate demand externalities and deleveraging. Rognlie, Shleifer, and Simsek (2018) suggest another channel involving investment overhang, whereby a glut of assets during the boom such as housing dampens the production of new assets after the crash. The latter is tricky to capture in our setup, since we assume assets are either endowed or created at date 0 but not thereafter.

booms represent bubbles, with the added caveat that they have evidence of underlying risk-shifting.

While our model suggests a role for intervention, the main remedies against booms policymakers have considered turn out to be ambiguous for welfare. This is because agents in our model can undertake two types of investments, speculation and a safe activity, and policy interventions affect both investments rather than just discourage speculation. For example, we find tighter monetary policy alleviates excessive leverage but suppresses productive investment and exacerbates misallocation. We show this can be mitigated if rather than tightening immediately, policymakers promise to tighten only if the boom continues (and, by implication, ease if it ends). We also find that macroprudential regulation can be counterproductive if restricting leverage particularly discourages less risky investments, leaving more resources for speculation. But unlike with monetary tightening, promising to restrict future leverage only makes things worse. Our takeaway is that policies for fighting booms must be carefully designed to ensure they disproportionately deter speculation. Successful intervention can involve monetary tightening or leverage restrictions, although the two are not equivalent: Tighter money raises interest rates, while leverage restrictions reduce demand for credit and lower interest rates. This offers a contrast to recent work by Caballero and Simsek (2019) that emphasizes aspects monetary tightening and leverage restrictions have in common.

The paper is organized as follows. Section 1 introduces the basic setup, focusing on a simple case where assets are riskless. We build on this framework in Section 2 to study risky assets, and show these give rise to asset booms and, in some cases, bubbles. Section 3 explores how the equilibrium of our model is inefficient and admits a role for policy intervention. Section 4 considers monetary policy and Section 5 considers macroprudential regulation, specifically restrictions on leverage. Section 6 concludes.

1 Credit, Production, and Assets

Our analysis requires a framework with credit, production, and assets. We begin with the simple case where the asset is riskless. In the next section we build on this setup and allow for risky assets. It is in the latter case where credit and asset booms emerge.

Consider an overlapping generations economy where agents live for two periods and only value consumption when old. That is, agents born at date t value consumption c_t and c_{t+1} at dates t and $t + 1$ at

$$u(c_t, c_{t+1}) = c_{t+1} \tag{1}$$

There is a fixed supply of identical assets normalized to one. For now, we assume these already exist at date 0, although later on we will consider the case where they must be produced. Each asset yields a constant real dividend $d > 0$ per period. In the next section we will allow the dividend to be stochastic.

There is a cohort of old agents at date 0 who start out owning all the assets. At each $t = 0, 1, 2, \dots$ a new cohort of agents is born. A cohort consists of two types. The first, whom we call savers, are endowed with

an aggregate e units of the good when young. They cannot produce or store goods, and must either buy assets or trade intertemporally to convert their endowment into consumption when old. The second type, whom we call entrepreneurs, can convert a good at date t into $1 + y$ goods at date $t + 1$ where $y > 0$, but only up to a finite capacity of one unit of input. Each entrepreneur is endowed with $w < 1$ goods while young. Since this is below their productive capacity, there is scope for savers and entrepreneurs to trade.

In principle, w and y can vary across entrepreneurs. For most of the analysis, we assume $w = 0$ for all entrepreneurs, so they must borrow all of their inputs. As will become clear in Section 5 when we allow for $w > 0$, allowing entrepreneurs to have wealth greatly complicates the analysis even though the qualitative results are unchanged. We do assume y varies across entrepreneurs. Let $n(y)$ denote the density of entrepreneurs with productivity y . We assume $n(y) > 0$ for all $0 \leq y < \infty$ and

$$e < \int_0^\infty n(y) dy < \infty \quad (2)$$

Condition (2) implies entrepreneurs collectively require more inputs than savers are endowed with.

Each period, then, savers must allocate their endowment e between buying assets and funding production by entrepreneurs to determine what they can consume when old. However, we assume trade between savers and entrepreneurs is subject to several frictions. At this point, when dividends are deterministic, these frictions are largely irrelevant. But once we allow for stochastic dividends in Section 2, they will matter.

1. **Transaction Costs:** Agents incur a fixed utility cost ϕ to trade with savers, where we let $\phi \rightarrow 0$.
2. **Information Frictions:** Savers cannot monitor if the agents they fund buy assets or produce. They also cannot observe any of the agent's wealth beyond the particular project the lender finances.
3. **Contracting Frictions:** Trade is restricted to non-contingent debt contracts, i.e., for each unit of funding agents receive at date t they must pay a fixed amount $1 + R_t$ at date $t + 1$.
4. **Default Costs:** If borrowers fail to pay their obligation, lenders can collect any proceeds from the project borrowers invested in, but the seizure wastes Φ resources per unit invested in the project.

The transactions cost ϕ ensures agents will not borrow for ventures that will lead them to default with certainty. We take the limit as $\phi \rightarrow 0$ to avoid keeping track of this cost. These costs eliminate equilibria in which agents borrow for strictly unprofitable purposes out of sheer indifference.

The information frictions we assume imply that although savers want to finance entrepreneurial production, they cannot prevent their borrowers from buying assets instead. With deterministic dividends, this will not be an issue, since buying assets will be unprofitable given transaction costs. But when we allow for stochastic dividends in the next section, some borrowers might buy assets instead of producing. While in our model lenders cannot tell whether their borrowers are producing or buying assets, we view this as a metaphor for situations in which all borrowing is used to buy assets, but lenders cannot tell the risk they face

from any given borrower or asset. For example, with new technologies, lenders cannot distinguish workable applications of new assets from speculative ventures. As another example, mortgage lenders cannot distinguish illiquid agents who value homeownership and earn a surplus from borrowing, much as entrepreneurs in our model earn $1 + y$, and speculators who earn rents and capital gains from buying houses but would default if prices fell.³ Assuming wealth is unobservable implies borrowers face limited liability, since lenders can only go after the resources they know about. Essentially, agents can borrow using non-recourse loans, or, alternatively, via shell entities that limit their liability to the returns on the project they borrow for.

The contracting frictions we assume are motivated by the empirical prevalence of non-contingent debt. However, this restriction on contracts plays an important role in preventing savers from screening borrowers who intend to speculate. When we allow for stochastic dividends in the next section, savers could discourage speculation by stipulating a higher repayment when the return on the asset is high and a lower one when the return is low. Imposing noncontingent debt rules out such arrangements. While we do not model the friction on contracting, we implicitly view it as a high cost to a third party of verifying contingencies.

Finally, our assumption that default costs are proportional to the scale of the project and not to the amount agents borrow captures the idea that auditing a borrower requires inspecting their projects. Again, although we model these as recovery costs, we view them as a stand-in for various costs and mechanisms that lead to diminished output when asset prices collapse.

An equilibrium consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$ that ensure both asset and credit markets clear when agents act optimally. To facilitate our exposition, suppose equilibrium prices p_t and interest rates R_t are deterministic. We confirm this is true in Appendix A. To solve for an equilibrium, we need supply and demand for assets and for credit. These can be easily characterized. Agents in their last period of life neither supply nor demand credit. They do own all assets, though, and will sell them if the asset price $p_t > 0$. Young savers are the only agents who can lend. They compare the return to lending $1 + R_t$ with the return to the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ and invest in whatever offers the highest return. Finally, young entrepreneurs choose whether to borrow to produce, and all young agents must choose whether to borrow to buy assets. Agents will borrow for any activity they expect to profit from. This means entrepreneurs will find it profitable to borrow to produce if and only if their productivity $y \geq R_t + \phi$. In the limit as $\phi \rightarrow 0$, entrepreneurs will produce iff their productivity $y \geq R_t$.

Savers use their endowment to either buy assets or make loans. Their borrowers will then either produce or buy assets. Hence, the endowment is ultimately used either to finance production or buy assets, implying

$$\int_{R_t}^{\infty} n(y) dy + p_t = e \tag{3}$$

Since we assume $n(y) > 0$ for all $y \geq 0$, there is a unique interest rate $R_t = \rho(p_t)$ that satisfies (3) for any

³Of course, the surplus agents who value homeownership obtain are not constant and depend on the current price of housing. For an example of a proper risk-shifting model of housing, see Barlevy and Fisher (2018).

asset price p_t , where $\rho(p_t)$ is increasing in p_t . Intuitively, a higher p_t reduces the amount of goods available for productive investment, so the interest rate on loans R_t must rise to lower demand from entrepreneurs.

Next, we argue the equilibrium interest rate on loans $1 + R_t$ must equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$. Suppose $R_t < r_t$. Agents could earn positive profits by borrowing to buy assets, since even with a fixed cost $\phi > 0$ borrowing would be profitable at a large enough scale. Demand for borrowing would then be infinite, yet the supply of credit is at most e , so this cannot be an equilibrium. Suppose instead that $R_t > r_t$. Savers would then earn more from lending than from buying the asset, so they would refuse to buy assets. Nor would any agent borrow to buy the asset, knowing she would default. Since the old sell the asset whenever its price is positive, this would require $p_t \leq 0$. But if the price were nonpositive, demand for the asset would be infinite. For both the credit and asset market to clear, then, we must have

$$1 + R_t = \frac{d + p_{t+1}}{p_t} = 1 + r_t \quad (4)$$

Note that (4) holds for any value of ϕ . When $\phi > 0$, no agent would borrow to buy assets that offer the same return as the interest rates on loans given the transaction costs involved. Taking the limit as $\phi \rightarrow 0$ implies agents will not borrow to buy assets in equilibrium. Substituting (3) into (4) implies

$$\begin{aligned} p_{t+1} &= (1 + \rho(p_t)) p_t - d \\ &\equiv \psi(p_t) \end{aligned} \quad (5)$$

where $\psi'(p_t) > 1$, $\psi(0) = -d < 0$, and $\lim_{p \rightarrow \infty} \psi(p) > e$. The graph of $\psi(p)$ is illustrated in Figure 1 together with the 45° line. The two lines intersect at the unique value p^d at which $p^d = \psi(p^d)$. For any initial condition, the law of motion $p_{t+1} = \psi(p_t)$ defines a unique path of asset prices. For any initial condition other than $p_0 = p^d$, the path will reach in finite time a value that is either negative or exceeds e , neither of which can be an equilibrium. Hence, the unique deterministic equilibrium is $p_t = p^d$ and $R_t = \rho(p^d) \equiv R^d$ for all t . Substituting $p_t = p_{t+1} = p^d$ in the zero-profit condition (4), implies

$$d = \rho(p^d) p^d$$

The right hand side is increasing in p^d . It follows that the equilibrium price p^d is increasing in d . Graphically, a larger d will shift the curve $p_{t+1} = \psi(p_t)$ in Figure 1 down, and so the steady state p^d will rise.

In Appendix A, we confirm $p_t = p^d$ is the unique equilibrium for this economy, implying the following:

Proposition 1 *When $d_t = d$ for all t , in the limit as $\phi \rightarrow 0$, the unique equilibrium features a constant price $p_t = p^d$ and constant interest rate $R_t = \rho(p^d) = R^d$. Only entrepreneurs with productivity $y > R^d$ produce, agents borrow only to produce and not to buy assets, and only savers hold assets,*

In equilibrium, the return on assets and loans are equal. Denote the common return to both activities by $R^d = \rho(p^d)$. Consider the present value of dividends discounted at this return. This is given by

$$f_t \equiv \sum_{j=1}^{\infty} \left(\frac{1}{1+R^d} \right)^j d = d/R^d = p^d$$

The value of dividends discounted at the return agents earn on their savings coincides with the price of the asset. When $d_t = d$ for all t , the asset will not be associated with a bubble. In short, when the asset is riskless, agents only borrow to produce and assets are properly priced. This will offer a contrast to what happens in the next section when we assume dividends are stochastic.

Remark 1: We can easily allow for multiple riskless assets. Suppose there were J assets indexed $j = 1, \dots, J$, each with fixed supply of 1 but potentially different fixed dividends d_j . Let p_{jt} denote the price of the j -th asset at date t . Define $d \equiv \sum_{j=1}^J d_j$ as the total dividends from all J assets and $p_t \equiv \sum_{j=1}^J p_{jt}$ as the value of all J assets. Resources that don't finance production will be used to buy assets, so (3) continues to hold. In addition, the return on each asset $1 + r_{jt} \equiv \frac{d_j + p_{j,t+1}}{p_{jt}}$ must equal the interest rate on loans $1 + R_t$. Combining these equalities implies (4). Hence, the equilibrium conditions for p_t and R_t are unchanged, but p_t now represents the total value of all assets, each of which offers the same return R_t . ■

Remark 2: With some modifications, we can also allow for a growing set of assets. This will be relevant in the next section, where we will argue that the periodic arrival of new types of assets can trigger asset booms. Suppose each period's old are endowed with a stock of new assets normalized to 1. Assets pay dividends one period after arrival. For aggregate dividends to remain constant, dividends on any single asset must decay over time. Let d_{st} denote the dividend at date t on assets that arrived at date s , and set

$$d_{st} = \begin{cases} (1 - \theta)^{t-1} d & \text{if } s = 0 \\ (1 - \theta)^{t-(s+1)} \theta d & \text{if } s = 1, 2, 3, \dots \end{cases} \quad \text{for } t \geq s + 1$$

By design, total dividends $\sum_{s=0}^{t-1} d_{st}$ in each period t sum to d . Let p_{st} denote the date- t price of the asset that arrived at date s , and set $p_t = \sum_{s=0}^t p_{st}$ as the total value of all assets around at date t . The market clearing condition (3) is unchanged. The return on each asset $1 + r_{st} \equiv \frac{d_{s,t+1} + p_{s,t+1}}{p_{st}}$ will equal the interest rate on loans $1 + R_t$. Aggregating over all assets available at date t yields the following alternative to (4):

$$1 + R_t = \frac{d + (p_{t+1} - p_{t+1,t+1})}{p_t}$$

The equilibrium value of all assets p_t will be constant and equal to $\frac{d}{R^d + \theta}$, where R^d denotes the equilibrium interest rate on loans. The price of any individual asset equals $p_{st} = \frac{d_{s,t+1}}{d} p_t = \frac{d_{s,t+1}}{R^d + \theta}$. ■

We conclude our discussion with a brief comment on welfare. In equilibrium, the amount savers spend on assets equates the return on the asset to the productivity of the marginal entrepreneur. Is this efficient? At first, it might seem that any resources spent on the asset are wasted, since the asset will yield d regardless of how much is spent on it while lending to entrepreneurs yields additional output. However, once we recognize that the asset must be held by someone between periods to ensure it survives from one period to the next, compensating the agents who hold the asset should be treated as an investment in safeguarding a stream of dividends for future cohorts. Efficiency dictates that the returns to all investments should be equal at the margin, and so equating the return to production and the return on the asset is in fact consistent with efficiency. In the next section, we show that returns will not be equated this way when assets are risky.

2 Risky Assets, Credit Booms, and Bubbles

We now turn to the case where dividends are stochastic. For this, we return to assuming there is only one type of asset. Let the dividend on this asset follow a regime-switching process such that the dividend d_t starts at $D > d$ when $t = 0$ and then switches to d with a constant probability $\pi \in (0, 1)$ in each period if it has yet to switch. Once the dividend falls to d , it will remain equal to d forever.

An equilibrium still consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and loan rates $\{R_t\}_{t=0}^{\infty}$. But since agents might now borrow both to buy assets and produce, we also need to track the share of lending used to buy assets, $\{\alpha_t\}_{t=0}^{\infty}$. These paths must still ensure asset and credit markets clear at all dates t and for any d_t .

In what follows, it will be convenient to distinguish for each date t whether d_t equals D or d . If $d_t = D$, agents who buy the asset at date t will be unsure about the dividend d_{t+1} they will receive at $t + 1$. If $d_t = d$, agents know the asset will pay a dividend of d at date $t + 1$. Let $(p_t^D, R_t^D, \alpha_t^D)$ denote an equilibrium if $d_t = D$ and $(p_t^d, R_t^d, \alpha_t^d)$ denote an equilibrium if $d_t = d$. Once dividends fall, the equilibrium will be as in Section 1, with $p_t^d = p^d$, $R_t^d = R^d$, and $\alpha_t^d = 0$ for all t . We only need to solve for $\{(p_t^D, R_t^D, \alpha_t^D)\}_{t=0}^{\infty}$.

We first show that we can solve for the equilibrium price p_t^D and interest rate on loans R_t^D independently of α_t^D . As before, savers allocate all of their endowment e either to finance production or to buy assets. The price p_t^D must thus continue to satisfy (3). Next, we argue that in equilibrium,

$$(1 + R_t^D) p_t^D = p_{t+1}^D + D \quad (6)$$

That is, the interest rate on loans $1 + R_t^D$ is equal to the return on the asset if $d_{t+1} = D$. We first argue that $p_{t+1}^D + D$ represents the maximum possible payoff to the asset, i.e., that

$$p_{t+1}^D + D > p^d + d$$

For suppose $p_{t+1}^D + D \leq p^d + d$. Since $D > d$, this requires $p_{t+1}^D < p^d$. From (3), we know the equilibrium interest rate on loans R_{t+1} must equal $\rho(p_{t+1})$. If $p_{t+1}^D < p^d$, then since $\rho'(\cdot) > 0$, we have

$$R_{t+1}^D = \rho(p_{t+1}^D) < \rho(p^d) = R^d$$

But then we would have

$$(1 + R_{t+1}^D) p_{t+1}^D < (1 + R^d) p^d = p^d + d.$$

This means that if $d_{t+1} = D$, an agent who borrows to buy assets at date $t + 1$ can make positive profits if $d_{t+2} = d$. But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. It follows that $p_{t+1}^D + D > p^d + d$.

To confirm (6), i.e., that the interest rate on loans is equal to the maximum return on the asset, suppose $(1 + R_t^D) p_t^D < p_{t+1}^D + D$. This would imply infinite demand for borrowing: Agents can earn positive profits if $d_{t+1} = D$ but default and earn zero if $d_{t+1} = d$. But the supply of credit is finite. Next, suppose

$(1 + R_t^D) p_t^D > p_{t+1}^D + D$. In this case, no agent would borrow to buy the asset knowing they would default and incur the cost ϕ . Since no agent borrows to buy assets, the only agents who borrow are entrepreneurs with productivity $y > R_t^D$, and they will repay for sure. The return to lending is R_t^D , which exceeds the expected return on the asset. No agent would buy the asset, so p_t^D would have to be nonpositive to ensure the old don't want to sell the asset. But this cannot be an equilibrium price, since if $p_t^D \leq 0$ there would be infinite demand for the asset. For both markets to clear, we need $(1 + R_t^D) p_t^D = p_{t+1}^D + D$.

Condition (6) is identical to the condition for an asset that offers a constant dividend $d_t = D$ for all t . From the previous section, we know there is a unique path $\{p_t^D, R_t^D\}_{t=0}^\infty$ that satisfies both this condition and (3). The equilibrium price p_t^D is thus constant and equal to p^D , where p^D solves

$$\rho(p^D) p^D = D$$

and the interest rate on loans R_t^D is given by $\rho(p^D) \equiv R^D$. This result is striking: The asset trades as if $d_{t+1} = D$ with certainty, even though d_{t+1} equals d with a probability π that can be arbitrarily close to 1.

The reason the asset is overpriced can be understood as follows. Lenders cannot monitor borrowers to insure they produce or use contingent contracts to screen out speculators, which allows agents to blend in with entrepreneurs and borrow to buy risky assets. Since borrowers can default and shift losses to creditors if the asset return is low, they only care about the maximal return on the asset. They will drive the price of the asset to the price consistent with the maximal dividend D , even if this realization is unlikely. We will turn to the question of whether this means the price exceeds the asset's fundamental value further below.

The only part of the equilibrium we still need to solve for is $\{\alpha_t^D\}_{t=0}^\infty$. For this, we use the expected return to the asset, denoted \bar{r}_t^D , and the expected return to lending, denoted \bar{R}_t^D . The former is given by

$$1 + \bar{r}_t^D = (1 - \pi) \left(1 + \frac{D}{p^D}\right) + \pi \left(\frac{d + p^d}{p^D}\right) \equiv 1 + \bar{r}^D \quad (7)$$

As for the expected return to lending, a fraction α_t^D of lending is used to buy assets and the rest finances production. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs, $1 + \bar{r}^D - \pi\Phi$. The remaining loans that finance production will be repaid in full, so the return on those loans is $1 + R^D$. This implies

$$\begin{aligned} 1 + \bar{R}_t^D &= (1 - \alpha_t^D) (1 + R^D) + \alpha_t^D (1 + \bar{r}^D - \pi\Phi) \\ &= (1 - \alpha_t^D) \left(1 + \frac{D}{p^D}\right) + \alpha_t^D (1 + \bar{r}^D - \pi\Phi) \end{aligned} \quad (8)$$

If $\bar{R}_t^D > \bar{r}^D$, savers would prefer lending over buying assets. The only agents who would buy assets would be those who borrow to do so, and so $\alpha_t^D = \frac{p^D}{e}$. If $\bar{R}_t^D = \bar{r}^D$, savers would be indifferent between buying assets and lending. This means α_t^D can assume any value between 0 and $\frac{p^D}{e}$. Finally, if $\bar{R}_t^D < \bar{r}^D$, savers would prefer buying assets over lending. No agent would borrow to buy assets, implying $\alpha_t^D = 0$. Hence, the expected return to lending \bar{R}_t^D and the share of lending used to buy assets α_t^D are jointly determined.

To solve for \bar{R}_t^D and α_t^D , consider first the case where $\alpha^D = \frac{p^D}{e}$. This can only be an equilibrium if $\bar{R}_t^D \geq \bar{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, i.e., only if

$$\left(1 - \frac{p^D}{e}\right) \frac{D}{p^D} + \frac{p^D}{e} (\bar{r}^D - \pi\Phi) \geq \bar{r}^D$$

Rearranging this equation and substituting in for \bar{r}^D implies $\alpha_t^D = \frac{p^D}{e}$ is an equilibrium only if

$$\Phi \leq \left(\frac{e}{p^D} - 1\right) \left(\frac{D+p^D-d-p^d}{p^D}\right) \equiv \Phi^* \quad (9)$$

Next, consider the case where $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$. This can only be an equilibrium if $\bar{R}_t^D = r^D$ when we evaluate \bar{R}_t^D at the relevant α_t^D . Since \bar{R}_t^D is decreasing in α_t^D , this requires that $\bar{R}_t^D < \bar{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, or

$$\Phi > \Phi^* \quad (10)$$

In this case, the equilibrium value of α_t^D is the one that equates \bar{R}_t^D and \bar{r}^D , which implies

$$\alpha_t^D = \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} \quad (11)$$

Finally, there cannot be an equilibrium in which $\alpha_t^D = 0$. This would require $\bar{R}_t^D \leq \bar{r}^D$ when $\alpha_t^D = 0$. But $\alpha_t^D = 0$ implies $\bar{R}_t^D = \frac{D}{p^D} > \bar{r}^D$. Hence, the value of α_t^D is unique and is either equal to $\frac{p^D}{e}$ or some value between 0 and $\frac{p^D}{e}$, depending on the cost of default Φ . We can summarize this result as follows:

Proposition 2 *When the dividend process follows a regime-switching process, in the limit as $\phi \rightarrow 0$, the unique equilibrium is given by*

$$(p_t, R_t) = \begin{cases} (p^D, R^D) & \text{if } d_t = D \\ (p^d, R^d) & \text{if } d_t = d \end{cases}$$

The share of lending used to buy assets α_t when $d_t = d$ equals 0 and when $d_t = D$ is given by

$$\alpha_t = \alpha^D = \begin{cases} \frac{p^D}{e} & \text{if } \Phi \leq \Phi^* \\ \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} & \text{if } \Phi > \Phi^* \end{cases}$$

Since $\alpha^D > 0$, it follows that some agents must borrow to buy assets when $d_t = D$. If all borrowing was for production, lending would entail no default risk. But given the equilibrium interest rate on loans R^D equals the maximum return on the asset, this would mean lending is more profitable than buying the asset. No agent would buy the asset, even as the old try to sell their assets, which cannot be an equilibrium. So borrowers have to buy up some assets. Which agents borrow to speculate is indeterminate. It could be entrepreneurs with $y < R^D$ who otherwise consume nothing, but our assumptions imply savers and productive entrepreneurs could also speculate without risking their net worth. In contrast to the case of $d_t = d$ in Section 1, lending now finances both production and speculation, and the asset seems overpriced.

Remark 3: In our model, there are no safe assets when $d_t = D$. Allowing for safe assets offering a return r^f no higher than the expected return on lending \bar{R}^D would not undermine our results. But if we allowed

for safe assets offering a return r^f above the value of \bar{R}^D in Proposition 2, savers would shift from lending to buying safe assets, depressing the price of the risky asset p^D and increasing the expected return on loans \bar{R}^D . This mirrors what we find in Section 4 when we allow agents to make deposits with a central bank: If the central bank offered to pay a higher rate on (safe) deposits, lending and the asset price p^D decline. The fact that a boom hinges on the absence of safe assets or a low return on such assets is reminiscent of work by Aoki, Nakajima, and Nikolov (2014), Caballero and Farhi (2017), and Acharya and Dogra (2018) on how an absence or shortage of safe assets can lead to bubbles. But the mechanism is different. In those papers, bubbles arise when the interest rate falls below the economy’s growth rate. Here, a low r^f encourages savers to “search for yield” and tolerate risky lending, similarly to Martinez-Meira and Repullo (2017). ■

We now argue our model captures key features of the booms in Borio and Lowe (2002), Jorda, Schularick, and Taylor (2015), and Mian, Sufi, and Verner (2017). That is, we show the equilibrium involves asset and credit booms, can involve bubbles, features high realized returns to savings even as borrowing is relatively cheap, and that the asset boom it features ends with a crash in asset prices and costly defaults.

Asset Price Booms: We begin with asset prices. The equilibrium price of the asset while $d_t = D$ will be the same as in an economy in which dividends remain equal to D forever. As we noted earlier, the price of an asset with a fixed dividend is increasing in the value of the dividend, so $p^D > p^d$. Our economy starts with a high asset price that collapses when dividends fall.

This equilibrium arguably fails to capture empirical asset booms. The latter feature rapid asset price growth with unchanging dividends, as opposed to high but stable prices and high dividends. However, we can generate a more realistic boom if we allowed dividends in the initial regime to start at d at date 0 and rise to D only if the initial regime survives long enough. Formally, suppose there is some finite date T such that, as long as we stay in the initial regime, $d_t = d$ until date T and $d_t = D$ for $t \geq T$. Once we switch regimes, dividends equal d forever.⁴ This specification accords with how new technologies promise eventual rather than immediate profits, and how rents in boom markets are initially stable even as house prices surge. If the regime remained unchanged through date T , the equilibrium from date T on would be as in Proposition 2. Between dates 0 and T , the equilibrium path of prices $\{p_t^D\}_{t=0}^T$ satisfy the law of motion

$$p_{t+1}^D = (1 + \rho(p_t^D)) p_t^D - d \equiv \psi^d(p_t^D)$$

with the boundary condition that $p_T^D = p^D$. We can use Figure 1 to solve for the path consistent with this boundary condition. Essentially, since $p^D > p^d$, the price p_t^D must start above p^d at date 0 and rise towards p^D at date T . The trajectory for the price p_t^D conditional on staying in the initial regime is given in Figure 2. The asset price follows an explosive path that grows at an increasing rate that exceeds the expected return to saving \bar{R}_t^D , even as dividends remain constant. The potential for high dividends that might be realized in the future fuels the growth in asset prices. Thus, our framework can generate more

⁴This setup is related to Zeira (1999). He assumed dividends grow until a stochastic date. In both his setup and ours, dividends rise more the longer the initial regime survives.

realistic asset booms and not just a high constant price, but we would then need to solve for an entire price path $\{p_t\}_{t=0}^T$. For analytical convenience, we will continue to assume d_t is constant within each regime.

Our setup also abstracts from how asset booms start. One might have thought we could start in the low dividend regime and transit to a temporarily high regime with some probability. But in that case, agents in the initial low regime would still borrow to buy assets, gambling that the high dividend regime would start next period. If dividends equal D throughout the high regime, the asset would trade at p^D in both the initial and temporarily high regime, and the boom would start before dividends rise. This is reminiscent of the Diba and Grossman (1987) result that asset bubbles are present from the very inception of the asset. Martin and Ventura (2012) show one can get around this result by allowing for the arrival of new assets that cannot be traded beforehand and let bubbles be associated with new assets. We could similarly allow for new assets as per Remark 2. Most new assets would pay a predictable but decaying return. Periodically, though, new assets arrive that start with temporarily high dividends.⁵ With some modifications, then, our model can allow for periodic booms. Again, for simplicity we do not pursue this approach here.

Credit Booms: Next, we show that the asset boom coincides with a boom in borrowing against assets. When $d_t = D$, the amount agents borrow to buy assets is given by

$$\frac{\alpha^D}{1 - \alpha^D} \int_{R^D}^{\infty} n(y) dy \quad (12)$$

By contrast, no one borrows to buy assets when $d_t = d$.

Since informational frictions imply it is hard to distinguish between borrowing to buy assets and borrowing for productive purposes, arguably the relevant empirical measure is not borrowing against assets but total borrowing. The total amount agents borrow to buy assets or produce is given by

$$\frac{1}{1 - \alpha_t} \int_{R_t}^{\infty} n(y) dy$$

Since $\alpha^D > 0$ while $\alpha^d = 0$, the term $\frac{1}{1 - \alpha_t}$ is higher during the boom. At the same time, with $R^D > R^d$, the integral $\int_{R_t}^{\infty} n(y) dy$ is smaller when $d_t = D$ than when $d_t = d$. Total lending can therefore rise or fall when d_t falls to d . When $\Phi \leq \Phi^*$, savers lend out all of their endowment e when $d_t = D$ but lend less than e when $d_t = d$. Only when $\Phi \gg \Phi^*$ can total borrowing fall with dividends. The asset boom is therefore associated with a boom in lending against assets, and, when Φ is small, a boom in total lending.

Asset Bubbles: We next turn to whether the boom in our model can be viewed as a bubble, in the sense that the price of the asset exceeds the present expected discounted value of its dividends. This question is hard to answer empirically. But overvaluation is often used to justify the need for policy intervention. We can compute the fundamental value of the asset in our model and ask whether the asset price indeed exceeds fundamentals and whether this condition indeed determines whether intervention is warranted.

⁵To ensure the return on assets is riskless outside of booms may require one-off changes in the dividends of existing assets if the assets that arrive are risky to ensure the return on existing assets is the same as it would be if the new assets were riskless.

We begin by defining the fundamental value of the asset in our model. For this, it will help to distinguish several rates of return when $d_t = D$. The first is the interest rate on loans R^D that borrowers are asked to repay. Recall R^D is equal to the maximal return on the asset, i.e.,

$$1 + R^D = 1 + \frac{D}{p^D} \quad (13)$$

During the boom, lenders will not expect to collect this interest in full, since a fraction $\alpha^D > 0$ of lending is used to buy assets and may result in default. Instead, lenders expect to earn $1 + \bar{R}^D$ defined as

$$1 + \bar{R}^D = (1 - \alpha^D \pi) \left(1 + \frac{D}{p^D}\right) + \alpha^D \pi \left(\frac{d+p^d}{p^D} - \Phi\right) \quad (14)$$

Finally, the expected return to buying the asset is given by

$$1 + \bar{r}^D = \frac{(1-\pi)(D+p^D) + \pi(d+p^d)}{p^D} \quad (15)$$

These three returns can be ranked, with $R^D > \bar{R}^D \geq \bar{r}^D$. The last inequality follows from the fact that if the expected return to buying the asset \bar{r}^D exceeded \bar{R}^D , agents would prefer to buy assets than lend. But demand for credit by entrepreneurs is positive at any finite interest rate, so this cannot be an equilibrium.

We need to take a stand on which rate to discount dividends when defining the fundamental value. Arguably, the relevant discount rate is the one at which agents would be willing to trade intertemporally. If an agent had a unit of resources, the best she can expect to earn on it is \bar{R}^D since in equilibrium she can do no better than lending out her unit. We therefore use \bar{R}^D as our discount rate. Since the equilibrium is stationary, the fundamental value of the asset f^D satisfies the recursive equation

$$f^D = \frac{\pi(d+p^d) + (1-\pi)(D+f^D)}{1+\bar{R}^D} \quad (16)$$

Equation (16) uses $1 + \bar{R}^D$ as the discount rate and the fact there $p^d = f^d$, since we argued in Section 1 that the price p^d coincides with the fundamental value d/R^d . Rearranging (16) implies

$$1 + \bar{R}^D = \frac{\pi(d+p^d) + (1-\pi)(D+f^D)}{f^D} \quad (17)$$

Comparing (17) with (15) shows that $p^D > f^D$ whenever $\bar{R}^D > \bar{r}^D$ and $p^D = f^D$ whenever $\bar{R}^D = \bar{r}^D$. The discussion of how to solve for α^D and \bar{R}^D that precedes Proposition 2 establishes that for $\Phi \geq \Phi^*$, the expected return on loans \bar{R}^D must equal the expected return on the asset \bar{r}^D . In this case the price of the asset coincides with fundamentals. But the same analysis implies that when $\Phi < \Phi^*$, the expected return on loans \bar{R}^D strictly exceeds the expected return on the asset \bar{r}^D . In this case, the asset price exceeds fundamentals. Whether a bubble exists therefore depends on Φ :

Proposition 3 *Let f^D denote the value of dividends discounted at the expected return on loans \bar{R}_t . Then the difference between the price of the asset and its fundamental value $b^D = p^D - f^D$ is*

$$b^D = (\pi(d+p^d) + (1-\pi)D) \left[\frac{1}{\pi + \bar{r}^D} - \frac{1}{\pi + \bar{R}^D} \right] \quad (18)$$

There bubble b^D is positive when $\Phi < \Phi^$ but equal to 0 when $\Phi \geq \Phi^*$.*

The fact that the asset is priced as if $d_{t+1} = D$ with certainty does not on its own imply the price exceeds fundamentals. Intuitively, bubbles arise in our model when leveraged agents who only care about the upside potential of the asset are willing to pay more than its expected value to buy it. When Φ is small, savers prefer to lend, and it is only leveraged agents who buy assets. In that case, the price of the asset will exceed fundamentals. When Φ is large, lending against the entire stock of assets is too costly, and in equilibrium savers will have to buy some of the assets. But savers who invest their own funds will refuse to pay more than fundamentals, so a bubble cannot occur. Previous work on risk-shifting which ignored default costs has tended to conflate risk-shifting with bubbles. But risk-shifting does not necessarily imply bubbles.

Although bubbles only arise when $\Phi < \Phi^*$, there is a sense in which agents spend too much on assets regardless of Φ . To see this, note that since $R^D > \bar{R}^D \geq \bar{r}^D$, the return R^D that the marginal entrepreneur can earn exceeds the expected return \bar{r}^D on the asset, regardless of whether $\bar{R}^D > \bar{r}^D$ or $\bar{R}^D = \bar{r}^D$. The asset thus yields too low a return compared to what entrepreneurs can achieve, meaning its price is too high. The price is too high not in the sense that it exceeds the present discounted value of future earnings, but that it only reflects the private return on the asset to the borrower. Whether the price of the asset exceeds fundamentals is arguably not the relevant concern for welfare. We return to this point in Section 3.

Realized Returns and Interest Rates: We next consider rates of return during the boom. Since $R^D > R^d$, the realized return on investment, both for those who buy assets and for those who lend, will be higher while the boom lasts. A boom will appear to be a good time for savers.

But even as *realized* returns must be higher during the boom, *expected* returns can be lower. The expected return to lending is \bar{R}^D during the boom and R^d after the boom. The expected return \bar{R}^D defined in (14) is a weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D} - \Phi$. Since $D/p^D = R^D > R^d$ and $R^d = d/p^d > d/p^D$, we have

$$1 + \frac{D}{p^D} > 1 + R^d > \frac{d+p^d}{p^D}$$

If the weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D}$ gives enough weight to the latter, e.g. if π is close to 1, the expected return to lending will be below $1 + R^d$ even before accounting for default costs. Asset booms can therefore be times of high realized returns but low expected returns.

In addition, even as savers earn higher realized returns during the boom, the interest rate at which agents borrow to buy risky assets is in an important sense too low. This is because when lenders cannot distinguish safe and risky borrowers, the former end up cross-subsidizing the latter. We can formalize this intuition by comparing the equilibrium in Proposition 2 to a hypothetical full-information benchmark. With complete information, lenders would charge those who buy assets an interest rate at least as high as the maximum return on the asset, $1 + D/\hat{p}^D$, where a hat denotes the asset price in full-information benchmark. At the same time, they would charge entrepreneurs an interest rate equal to the expected return on the asset $(1 - \pi)(1 + D/\hat{p}^D) + \pi(d + p^d)/\hat{p}^D$, as opposed to the maximum return on the asset in Proposition 2. At this lower rate, entrepreneurs will borrow more under full information, leaving fewer resources to spend on

the asset. As a result, the asset price \hat{p}^D with full information will be lower than p^D , which implies

$$R^D = \frac{D}{p^D} < \frac{D}{\hat{p}^D}$$

In this sense, agents who borrow to buy risky assets are charged lower interest rates than they would to borrow equally risky assets that lenders can easily evaluate, and the interest rate R^D doesn't reflect the full risk of the assets speculators purchase.

Fallout from the Crash: Finally, we turn to how asset booms end in our model. When dividends fall, agents who previously borrowed to buy assets will be forced to default. This imposes a cost of Φp^D on lenders. The collapse in asset prices thus triggers a fall in the resources this cohort can consume, above and beyond the decline in the dividend income they earn. By construction, the decline is proportional to the price of assets p^D during the boom. A larger boom thus implies a larger loss once the boom ends. In our model this is because recovery costs are larger when more resources are invested in assets. But, as we noted above, we view this as a stand-in for other channels in which a fall in asset prices would lead to lower output, e.g. debt overhang and or deleveraging. In all of these mechanisms, the decline in output after a crash increases in the size of the run-up in asset prices during the boom.

Our model can thus capture the asset booms and busts we see in practice. In the remainder of the paper, we examine whether in our model there is a reason to intervene against these booms, and whether the particular interventions policymakers have debated can in fact improve welfare.

3 Inefficiency of Equilibria

In this section, we argue that the asset boom in the high regime is inefficient in two distinct senses. The first concerns misallocation: The marginal return to production during the boom exceeds the expected return on assets, so there are gains to redirecting some resources spent on assets to production. The second concerns excessive leverage: Agents who borrow to buy assets take on too much debt because they ignore the default costs Φp^D they impose on others. These inefficiencies arise regardless of how Φ compares with Φ^* , i.e., regardless of whether asset prices exceed fundamentals. Given evidence of risk-shifting, policymakers would not need to determine if bubbles are present to decide whether to intervene.

We begin with misallocation. As we noted above, when $d_t = D$, the productivity of the marginal entrepreneur is equal to the interest rate on loans R^D . But R^D exceeds the expected return on loans \bar{R}^D , since agents who borrow to buy assets can default. At the same time, \bar{R}^D is at least as high as the expected return on the asset \bar{r}^D or else savers would refuse to lend, which cannot be an equilibrium given demand for credit from entrepreneurs is positive for any finite interest rate. Hence, $R^D > \bar{r}^D$, which implies young agents could achieve a higher return on their endowment if they shifted some of what they spend on assets to the marginal entrepreneur. Intuitively, agents who borrow to buy assets ignore the losses their lenders incur. As a result, their private gain to buying the asset exceeds its social return, and too many resources

are allocated to buying assets. Once the boom ends, the return on the asset d/p^d will be the same as the productivity of the marginal entrepreneur R^d and resources will be allocated efficiently.

While a given cohort could secure a higher return by coordinating to shift resources from buying assets to producing, doing so would hurt the previous cohort from whom they buy assets. Redirecting resources to production therefore does not achieve a strict Pareto improvement in our model as specified. A similar point was made in Grossman and Yanagawa (1993). They also studied an overlapping generations economy in which agents can use resources to either buy assets or produce. While their model did not feature risk shifting, it did feature a production externality that implies the return to production is higher than the return on the asset. One of their key results was that despite this, it is impossible to make all agents better off by reallocating resources. This is because the reallocation needed to improve how resources are allocated would hurt the original asset owners. We now argue that this impossibility hinges on assuming an exogenous supply of assets. If agents can create additional assets at a cost, as is certainly true for new technologies or housing, shifting resources from asset creation to entrepreneurs can make all agents better off.

Formally, suppose the old at date 0 are endowed with neither goods nor assets, but they know how to convert goods into assets. For simplicity, we assume assets can only be created at date 0. The technology for producing assets is summarized by an increasing function $c(q)$ which denotes the amount of goods needed to produce the q -th asset. This could be because old agents can each produce one asset but differ in productivity. Since only the young are endowed with goods, they must provide goods to the old to create assets. We assume the old collect the revenue from asset sales up front. They then use some of these goods to produce assets and consume any of the goods left over. Optimality dictates they should create assets up to the point q^* at which the marginal cost $c(q^*)$ equals the price p_0 . Hence, they will collect $p_0 q^*$ in revenue, which exceeds the amount of goods $C(q^*) \equiv \int_0^{q^*} c(q) dq$ needed to produce the q^* assets they sell.

With endogenous asset creation, equilibrium condition (6) remains unchanged. However, we need to replace p_t in (3) with $p_t q^* = p_t c^{-1}(p_0)$. The latter is increasing in p_t for all t , and so we can show that the equilibrium remains unique and qualitatively similar to before. Suppose we intervene and reduce the quantity of assets produced at date 0 at the margin from its equilibrium value q^* . Since $c(q^*) = p_0$, the consumption of the old will be unchanged. Cohorts born at each date $t \geq 0$ can use the resources they would have spent on the last asset for production. If $d_t = D$ at date t , the return on production R^D exceeds the expected return \bar{r}^D the asset would have delivered. If $d_t = d$ at date t , the return R^d on production would be the same as the return r^d the asset would have delivered. Since $\Pr(d_t = D) > 0$ for all t , reducing q^* would make all cohorts better off ex ante. The difference is that the intervention we are considering now involves creating fewer assets and not just providing certain agents with fewer rents.

When default costs $\Phi > 0$, a second inefficiency emerges that corresponds to excessive leverage. Even if we hold the supply of assets q^* created at date 0 fixed, agents could be made better off if lenders would directly buy the assets their borrowers purchase and reimburse those borrowers for the income they would have earned. This avoids the deadweight loss Φp^D that lenders incur when borrowers default. Essentially,

there is no socially useful purpose for agents to borrow and buy risky assets. But they do so in equilibrium because they don't bear the costs of their default. This same inefficiency would arise if Φ represented not recovery costs but forgone output when asset prices fall because of debt overhang or deleveraging.

While these inefficiencies arise for any $\Phi > 0$, Proposition 3 implies bubbles only arise when $\Phi < \Phi^*$. Thus, although bubbles can arise in our model, they are a symptom of an underlying distortion rather than the root problem. Risk-shifting is costly not because asset prices exceed fundamentals, but because they do not reflect the true social value of assets given the various externalities speculators impose on others. Policymakers who have evidence of risk-shifting should intervene whether asset prices equal fundamentals or not. The purpose of intervention is not to equate prices with fundamentals but to discourage speculation that leads to excessive asset creation and results in deadweight losses from default.

Since policymakers presumably face the same difficulties as lenders in distinguishing speculation from productive borrowing, they cannot simply discourage or ban speculation. They can, however, turn to blunt tools such as monetary policy or leverage restrictions that affect speculation as well as other investments. The remainder of the paper considers whether such interventions can still improve welfare.

To study these interventions, we will need to relax some of the simplifying assumptions we have relied on so far. First, to capture the effects of monetary policy, we must relax our assumption that each cohort is endowed with an exogenously fixed supply of goods e to allocate to production and assets. While this assumption is convenient, models of monetary policy often rely on price rigidities that allow economic activity to expand or contract when the monetary authority moves so that monetary policy matters. In the next section, we drop our assumption of a fixed endowment e to incorporate this mechanism.

To capture the effect of leverage restrictions, we will need to relax our assumption that entrepreneurs are endowed with nothing. When borrowers have no resources, there is no way to restrict leverage other than cutting credit off altogether. In Section 5, we return to assuming savers are endowed with a fixed amount of goods, but we assume entrepreneurs are also endowed with resources. While entrepreneurs without wealth must take on infinite leverage, those with wealth face a choice of how much leverage to take on. This introduces a complication we have managed to avoid so far, namely that we need a continuum of markets to span all possible choices of leverage agents might entertain. By contrast, when agents had no endowment, all credit could be intermediated in a single market. We discuss how to deal with the complication of a continuum of markets, and then study restrictions on the degree of leverage borrowers can take on.

4 Monetary Policy

This section explores monetary policy. As we noted above, this requires us to abandon our assumption that savers are endowed with an exogenous amount of goods. We follow Galí (2014), who also studies monetary policy in an overlapping generations economy with assets. First, we assume savers are endowed with labor

that can be used to produce goods rather than goods themselves. Second, we introduce a monetary authority and monopolistic competition so the suppliers who hire labor set the prices of their goods. We assume the monetary authority moves after goods producers set their prices but before they hire labor. This allows the real wage – and consequently output – to respond to monetary policy.

We leave the details of the analysis to Appendix B and only sketch the results here. Our assumptions imply labor supply only depends on the real wage. Under these assumptions, in the absence of money, the equilibrium real wage will be constant over time and independent of d_t . Thus, absent money, the reduced-form representation of our economy is the same as we have assumed up to now: Each cohort of savers has a constant budget e which it allocates between entrepreneurial activity and purchasing assets.

Next, we introduce a monetary authority that can announce a nominal interest rate at which it is willing to borrow and lend. As in Galí (2014), we consider an equilibrium in which money doesn't circulate. This requires inflation to adjust so that the real value of the nominal rate set by the monetary authority equals the real return agents earn elsewhere, leaving agents indifferent to holding money. At the beginning of each period, producers set the prices of the goods they expect to sell. The monetary authority then sets a nominal interest rate. Finally, producers hire workers and produce goods. If producers could perfectly anticipate what the monetary authority will do, the nominal interest rate will not affect the real wage or any other real variable: Producers will set their prices as a markup over the nominal wage they know will prevail, which implies the same real wage as in the absence of money and hence the same earnings e .

If producers cannot perfectly anticipate what the monetary authority will do, producers will set their price as a markup over the expected nominal wage that will prevail after the monetary authority moves. If the nominal interest rate this period turns out to be higher (lower) than expected, the nominal and real wage can be higher (lower) than expected. Essentially, an unanticipated move by the monetary authority allows a self-fulfilling fall in demand for goods. Lower demand for goods means producers don't need to hire as much labor, the real wage falls, and since agents earn less, demand for goods will indeed be lower. A surprise move by the monetary authority at date 0 can thus change earnings e_0 , just as an income tax or subsidy would. Since producers set prices at the beginning of each period, an intervention at date 0 will not affect real variables beyond date 0. We can therefore deduce the implications of such a policy on asset prices and interest rates using comparative statics on e_0 in our original endowment economy holding $e_t = e$ at all other dates. The next proposition, based on our analysis in Appendix B, summarizes these effects.

Proposition 4 *An unanticipated monetary contraction at date 0 that reduces earnings e_0 below the earnings e that would have prevailed absent any intervention leads to a lower asset price p_0^D and a higher real interest rate on loans R_0^D at date 0 than would have prevailed absent any intervention.*

We next turn to the welfare implications of a contractionary intervention at date 0. Since an intervention at date 0 will have no impact on real variables beyond date 0, cohorts born at dates $t = 1, 2, 3, \dots$ will be

unaffected. The cohort born at date 0 works less, and expects to consume the amount

$$[(1 - \pi)(D + p_1^D) + \pi(d + p_1^d)] - \pi\Phi p_0^D + \int_{R_0^D}^{\infty} (1 + y)n(y)dy \quad (19)$$

The first term in (19) represents the expected payout on the asset at date 1 and is unaffected by what the monetary authority does at date 0. The next term represents expected default costs. A contractionary policy at date 0 drives down the price p_0^D and lowers the expected costs of default. The last term represents the output entrepreneurs born at date 0 produce at date 1. Since tighter monetary policy increases R_0^D , fewer entrepreneurs produce. A contractionary monetary policy thus mitigates excessive borrowing against assets but exacerbates underproduction by entrepreneurs. The impact of the intervention on this cohort is ambiguous, but for sufficiently large Φ the first effect will dominate and this intervention would allow this cohort to consume more and work less. Finally, the old at date 0 will be worse off, since their earnings p_0^D fall. However, since the effect of policy on p_0^D and R_0^D is independent of Φ , for sufficiently large Φ it will be possible for the cohort at date 0 to leave the old at date 0 whole and still be better off on account of the lower default costs. Hence, for large Φ , a contractionary policy can be used to generate a Pareto improvement, at least if we can redistribute resources across agents. Our result is reminiscent of Svensson (2017), who examines the tradeoff between the cost of tighter monetary policy against the benefits of lowering the odds of a financial crisis. In our framework, the probability the boom ends is fixed at π , but tighter monetary policy mitigates the severity of the output decline if the boom ends. The gain from a lower expected fall in output at date 1 must be compared to the cost of impoverishing agents at date 0.

It turns out there is a better way to use monetary policy to intervene against an asset boom. Suppose the monetary authority did nothing at date 0 but credibly promised to be contractionary at date 1 if the boom continued, i.e., if $d_1 = D$. Here, we assume the dividend d_t is revealed after producers set their prices at the beginning of date t but before the monetary authority moves. Since producers at date 1 set prices based on the expected nominal wage as a function of the dividend d_1 , monetary policy can only be contractionary when $d_1 = D$ if it is also expansionary when $d_1 = d$. That is, the monetary authority intervenes to set $e_1^d > e$ if $d_1 = d$ and $e_1^D < e$ if $d_1 = D$. Per Proposition 4, this policy will depress p_1^D and increase R_1^D if $d_1 = D$. However, as we show in Appendix B, this intervention will lower both p_0^D and R_0^D at date 0.

Proposition 5 *A commitment by the monetary authority at date 0 to set $e_1^d > e > e_1^D$ leads to a lower asset price p_0^D and a lower interest rate on loans R_0^D at date 0 than would have prevailed absent any intervention.*

In contrast to a direct immediate intervention that exacerbates misallocation, a threat to tighten if the boom continues mitigates both excessive leverage and misallocation at date 0. This approach could raise welfare even when immediate tightening cannot. Formally, cohorts born at $t = 2, 3, \dots$ will be unaffected by an intervention at date 1. We show in Appendix B that this intervention will make the cohort born at date 1 better off if $d_1 = d$. Although they work more, the monopoly power we assume so that firms act as price setters implies inefficiently low employment in the absence of intervention, so higher employment is better. Whether the cohort born at date 1 will be better off if $d_1 = D$ is ambiguous, just as a direct intervention at

date 0 is: This cohort can fund less production by entrepreneurs but will incur smaller default costs Φp_1^D . Even when Φ is small, though, as long as the probability π that dividends fall is close to 1, this cohort can be made better off *ex ante*. The cohort born at date $t = 0$ will be strictly better off, as both expected default costs Φp_0^D are lower and more entrepreneurial activity is financed when R_0^D is lower. Finally, the old at date 0 will be worse off given they earn less from selling assets at a lower price p_0^D . But when $\Phi > 0$, the young at date 0 can compensate the old and remain better off.

The reason delayed intervention works better is that it is more targeted at speculators. Tightening at date 1 has no direct effect on the entrepreneurs who borrow at date 0 to produce, but it will lower the price speculators receive from selling the asset if the boom continues. Essentially, a state-contingent policy is substituting for the type of contingencies savers would want to incorporate if they weren't restricted to simple debt contracts. The way in which monetary policy is implemented matters for welfare.

5 Macprudential Regulation

We now turn to macroprudential policy. Intuitively, leverage restrictions reduce demand for credit, which lowers interest rates. Lower interest rates in turn tend to raise asset prices, especially if, as in our model, demand for credit falls more among entrepreneurs, freeing up resources for speculation. Tighter leverage restrictions thus have the opposite effect on interest rates and asset prices as tighter monetary policy. At the same time, leverage restrictions reduce the fraction of assets financed with borrowing. The overall effect of this intervention is thus ambiguous: Fewer assets are financed with debt, but agents spend more per asset. In contrast to tighter monetary policy, leverage restrictions may increase welfare losses from default if they raise total borrowing against assets. Our results offer a contrast to Caballero and Simsek (2019), who describe a different economy where leverage restrictions and tighter monetary policy are welfare equivalent.

To analyze leverage restrictions, we need to modify our assumption that entrepreneurs are endowed with nothing. When entrepreneurs lack all resources, any down-payment requirement would shut down credit. To analyze interventions that restrict rather than eliminate leverage, borrowers must be able to keep producing and speculating even when leverage is restricted. We therefore allow entrepreneurs to be endowed with resources. But this modification introduces its own complication. When agents had nothing, they had to be infinitely levered. Now they must choose how much leverage to take on. This requires multiple markets to accommodate all possible leverage choices rather than a single market as we have considered so far.

To keep the analysis tractable, we return to assuming agents are endowed with goods rather than labor. As before, each cohort consists of unproductive savers endowed with e total goods and entrepreneurs who can convert goods at date t into goods at date $t + 1$. But rather than assume entrepreneurs are equally endowed with $w = 0$ and differ in productivity y , we now consider the opposite case where entrepreneurs differ in endowments w and share the same productivity y^* . We discuss the case where both w and y vary across entrepreneurs at the end of this section.

We assume the wealth of entrepreneurs w is distributed uniformly. Specifically, for each $w \in [0, 1]$, there is a constant density $2\varphi e$ of entrepreneurs with wealth w , where e is the endowment of savers and φ is a constant such that $0 < \varphi < 1$. The combined endowment of all entrepreneurs is therefore

$$\int_0^1 w (2\varphi e) dw = \varphi e$$

The total endowment of savers and entrepreneurs is $(1 + \varphi) e$. To produce at capacity, entrepreneurs need

$$\int_0^1 (1 - w) (2\varphi e) dw = \varphi e$$

Since $\varphi < 1$, entrepreneurs require fewer resources than savers have, in contrast to what we assumed in (2).

We assume the common productivity y^* is large enough to exceed the maximal return on the asset. To establish that this maximal return is finite, observe that the asset price p_t is bounded above by $(1 + \varphi) e$, the most each cohort has to spend on the asset, and is bounded below by $(1 - \varphi) e$, the amount of resources left to spend on the asset if all entrepreneurs produce at capacity. The maximal return on the asset occurs when $d_{t+1} = D$, the price of the asset at date t assumes its lowest value $(1 - \varphi) e$, and the price at $t + 1$ assumes its maximum value $(1 + \varphi) e$. We assume $1 + y^*$ exceeds this return, i.e.,

$$1 + y^* > \frac{D + (1 + \varphi) e - (1 - \varphi) e}{(1 - \varphi) e} = \frac{D + 2\varphi e}{(1 - \varphi) e} \quad (20)$$

Assumption (20) ensures production dominates other investments in terms of return, so all entrepreneurs will want to produce at capacity in equilibrium. This allows us to avoid solving for the endogenous fraction of entrepreneurs funded in each of a continuum of markets, which greatly simplifies the analysis.

Since entrepreneurs have positive wealth, they can help finance their investments. We continue to assume lenders cannot observe what borrowers invest in, but they can observe the resources borrowers use to finance their project. Verifying that borrowers invest their own wealth is not the same as verifying what they invest in. A lender cannot observe any additional resources his borrower has beyond what she invests in her project. Essentially, borrowers choose how much to put in the shell entity they borrow through. By paying a share of their investment, the borrower discloses resources that the lender can go after in case of default.

Formally, borrowers choose the fraction $\lambda \in [0, 1)$ of their investment to finance. We model this as a continuum of markets indexed by $\lambda \in [0, 1)$. An agent who borrows in market λ can borrow $\frac{1-\lambda}{\lambda}$ units for each unit of her own wealth that she invests. She can thus leverage her endowment of w to finance an investment of size $\frac{w}{\lambda}$. When $w > 0$, the choice of leverage is non-trivial: By going to a market with a lower λ , an entrepreneur can borrow more and produce at a larger scale, but this will leave their lender with a smaller cushion to go after in case of default. Back when we assumed all entrepreneurs had no wealth, agents had no choice. They could only borrow in market $\lambda = 0$ and choose infinite leverage. Now that agents have wealth, we need a market for each $\lambda \in [0, 1)$ to accommodate any leverage they might choose. The reason we assumed entrepreneurs had no resources up to now is precisely to focus on a single market.

We now define and solve for an equilibrium when there is a continuum of markets. To anticipate where we are going, we describe an equilibrium in which entrepreneurs with wealth w go to market $\lambda = w$ and invest their entire endowment in production, borrowing $1 - w$ to produce at capacity. Entrepreneurs with different w thus sort into different markets. As before, when $d_t = D$, some agents will borrow to buy assets in equilibrium. However, they will only borrow in markets with low λ . This motivates us to consider macroprudential leverage restrictions that involve shutting down markets where λ is below some floor $\underline{\lambda}$.

5.1 Equilibrium with Multiple Markets

An equilibrium in our economy still consists of a path of asset prices $\{p_t\}_{t=0}^{\infty}$ and a path of interest rates, but the latter now consists of a path of interest rates $\{R_t(\lambda)\}_{t=0}^{\infty}$ for each market $\lambda \in [0, 1)$ and amounts borrowed in each market $\lambda \in [0, 1)$ for each purpose. Let $f_t^a(\lambda)$ and $f_t^p(\lambda)$ denote the rate at which agents borrow in market λ to buy assets and produce, respectively, and $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote total borrowing in market λ . We can integrate these rates to obtain the total amounts borrowed in all markets, $\int_0^1 f_t^a(\lambda) d\lambda$ and $\int_0^1 f_t^p(\lambda) d\lambda$. Although we refer to borrowing rates, we do not require agents to borrow infinitesimal amounts in all markets. Indeed, once we introduce leverage restrictions, there will be a market that will attract a positive mass of borrowers. We discuss how to deal with this formally in Appendix C, but, loosely, such markets can be viewed as having infinite borrowing rates. We refer to market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$. The price p_t , interest rates $R_t(\lambda)$, and amounts borrowed $f_t^a(\lambda)$ and $f_t^p(\lambda)$ must ensure markets clear when agents act optimally, just as with a single credit market.

To determine if lenders are optimizing, we need to know what they expect to earn from lending in any market $\lambda \in [0, 1)$. Building on our previous notation, let $\bar{R}_t(\lambda)$ denote the expected return to lending at date t in market λ . If market λ is active, the expected return $\bar{R}_t(\lambda)$ to lending in market λ must equal what lenders recover from the agents they lend to. We can thus deduce $\bar{R}_t(\lambda)$ from the interest rate $R_t(\lambda)$ and the amounts $f_t^a(\lambda)$ and $f_t^p(\lambda)$ agents borrow to buy assets and produce, respectively. But if market λ is inactive, there is nothing to guide lenders on what to expect if they were to lend to a market where no borrowers show up. Instead, we need to assign an expected return $\bar{R}_t(\lambda)$ to each inactive market as part of our definition of an equilibrium. In what follows, we first look for an equilibrium in which all markets are active to avoid the question of how to assign $\bar{R}_t(\lambda)$ in inactive markets. We then discuss equilibria in which markets can be inactive. This naturally leads into our analysis of regulatory interventions in which some markets are inactive by decree rather than because of what agents believe.

We begin with the case where $d_t = d$ for all t . As in Section 1, we proceed as if equilibrium prices are deterministic and verify this is the case in Appendix C. In this case there will be no default, and so the expected return to lending $\bar{R}_t(\lambda)$ will equal the interest rate on loans $R_t(\lambda)$ in each active market λ . The expected return in all active markets must be the same for lenders to agree to lend in all of these markets, and so $R_t(\lambda)$ must be the same in all active markets. Moreover, this common interest rate must equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ to ensure savers agree both to buy assets and to lend in active markets. That is, $R_t(\lambda) = r_t$ in all active markets λ . At these interest rates, borrowing to buy assets is unprofitable

given $\phi > 0$. Since (20) ensures y^* exceeds r_t , all entrepreneurs will want to borrow in order to produce at capacity. Given $R_t(\lambda)$ is the same for all λ , entrepreneurs will be indifferent as to which market they borrow in, as long as they borrow enough to reach capacity. This includes the case where those with wealth w borrow $1 - w$ in market $\lambda = w$, an arrangement that ensures all markets are active.

Since all entrepreneurs will produce at capacity, the amount invested in production is $2\varphi e$. Any of the total endowment $(1 + \varphi)e$ of each cohort not used to produce will be spent on the asset. This implies

$$p_t + 2\varphi e = (1 + \varphi)e \quad (21)$$

It follows that $p_t = (1 - \varphi)e$ for all t . The return to buying the asset r_t and the interest rate on loans $R_t(\lambda)$ in all markets λ will then be $\frac{d}{(1-\varphi)e}$. This leads to the following analog to our earlier Proposition 1:

Proposition 6 *When $d_t = d$ for all t , there exists an equilibrium in which all markets are active. In any such equilibrium, $p_t = (1 - \varphi)e \equiv p^d$ for all t , $R_t(\lambda) = \frac{d}{(1-\varphi)e} \equiv R^d$ for all $\lambda \in [0, 1]$ and all t , all entrepreneurs borrow and produce at capacity, no agents borrow to buy assets, and only savers hold assets.*

Next, we turn to the case where $d_t = D$ at date 0 and permanently switches to d with constant probability π per period. We again use a superscript D to refer to an equilibrium object at date t when $d_t = D$. We begin by solving for equilibrium interest rates. For each active market λ where $f_t(\lambda) > 0$, either agents borrow to buy assets, i.e., $f_t^a(\lambda) > 0$, or they do not, i.e., $f_t^a(\lambda) = 0$. In the latter case, there will be no default and the expected return to lending $\overline{R}_t^D(\lambda)$ will equal the interest rate on loans $R_t^D(\lambda)$. In equilibrium, $\overline{R}_t^D(\lambda)$ must be the same in all active markets for lenders to agree to lend in these markets. Denote this common expected return by \overline{R}_t^D . Then $R_t^D(\lambda) = \overline{R}_t^D$ in any active market λ in which $f_t^a(\lambda) = 0$.

Consider next an active market λ in which agents do borrow to buy assets, i.e., $f_t^a(\lambda) > 0$. Agents would only borrow to buy assets if they intend to default if the return on the asset is low. Borrowing to buy assets and not defaulting cannot be profitable, since lenders would not lend at an interest rate below the expected return they could earn from buying the assets themselves. As long as $\phi > 0$, the only way we can get agents to borrow to buy assets is if they default when the returns to the asset are low. The expected payoff from this strategy per unit spent to buy assets is given by

$$(1 - \pi) \left[\frac{p_{t+1}^D + D}{p_t^D} - (1 - \lambda)(1 + R_t^D(\lambda)) \right] \quad (22)$$

For each unit of resources agents spend on assets by borrowing in market λ , a fraction λ must come from their own wealth. If they had lent out this fraction instead, they would have earned $(1 + \overline{R}_t^D)\lambda$. We now argue that in equilibrium, this payoff must equal (22). If $(1 + \overline{R}_t^D)\lambda$ exceeded (22), nobody would borrow to buy assets in market λ given they could earn more from lending, contradicting the fact that $f_t^a(\lambda) > 0$. Conversely, if $(1 + \overline{R}_t^D)\lambda$ were lower than (22), no agent would be willing to lend in any market given they can borrow in market λ to buy assets, again contradicting the fact that $f_t^a(\lambda) > 0$. Equating the two payoffs yields an expression for the interest rate on loans $R_t^D(\lambda)$ in any active market λ in which $f_t^a(\lambda) > 0$:

$$1 + R_t^D(\lambda) = \frac{1}{1-\lambda} \left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda(1 + \overline{R}_t^D)}{1-\pi} \right] \quad (23)$$

Thus, we have expressions for the interest rate $R_t^D(\lambda)$ if $f_t^a(\lambda) = 0$ and if $f_t^a(\lambda) > 0$, respectively. The next lemma, derived in Appendix C, shows there exists a cutoff $\Lambda_t^D \in [0, 1)$ such that $R_t(\lambda)$ is given by (23) in markets $\lambda < \Lambda_t^D$ but is equal to \bar{R}_t^D in markets $\lambda \geq \Lambda_t^D$.

Lemma: *If all markets are active, then there exists a cutoff $\Lambda_t^D \in [0, 1)$ such that*

$$1 + R_t^D(\lambda) = \begin{cases} \frac{1}{1-\lambda} \left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda(1 + \bar{R}_t^D)}{1-\pi} \right] & \text{if } \lambda \in [0, \Lambda_t^D) \\ 1 + \bar{R}_t^D & \text{if } \lambda \in [\Lambda_t^D, 1) \end{cases} \quad (24)$$

Figure 3 plots the schedule of interest rates from (24). In market $\lambda = 0$, where agents are infinitely levered, the interest rate $R_t^D(0)$ equals the maximal return on the asset, $\frac{p_{t+1}^D + D}{p_t^D}$. This is the same as in Section 2, where $\lambda = 0$ was the only possible market. The logic is the same: When agents put no resources down, they must hand over all returns from the asset to the lender to ensure they earn no profits. For $0 < \lambda \leq \Lambda_t^D$, the interest rate $R_t(\lambda)$ decreases with λ . We prove this formally in Appendix C, but intuitively, when the borrower effectively pledges more of her own resources, the lender need not charge as much in interest to cover shortfalls in case of default. Finally, for $\lambda \geq \Lambda_t^D$ the interest rate $R_t(\lambda)$ is constant and equal to \bar{R}_t^D .

Figure 3 reveals that, in equilibrium, credit markets fall into two groups: In markets with $\lambda < \Lambda_t^D$ there is some borrowing to buy assets, while in markets with $\lambda \geq \Lambda_t^D$ agents only borrow to produce. We know this because in markets with $\lambda < \Lambda_t^D$ the interest rate $R_t^D(\lambda)$ exceeds the expected return \bar{R}_t^D lenders earn. If lenders are to earn \bar{R}_t^D , some borrowers in these markets would have to default. As an aside, for $\lambda > 0$, agents who buy risky assets must invest their own wealth in assets. They must therefore earn strict profits if $d_{t+1} = D$ to offset their losses if $d_{t+1} = d$. By contrast, in markets with $\lambda \geq \Lambda_t^D$, the interest rate on loans $R_t^D(\lambda) = \bar{R}_t^D$. For lenders to earn \bar{R}_t^D , all borrowers must repay in full. Since we know agents only borrow to buy assets if they intend to default, the absence of default means agents in these markets only borrow to produce. Intuitively, borrowers won't speculate if they have enough skin in the game.

Given the equilibrium interest rates schedule (24), we can now solve for what entrepreneurs do. Recall that (20) implies y^* exceeds the maximal return on the asset. We just argued $R_t^D(0)$ is equal to this maximal return, and that $R_t^D(0)$ exceeds \bar{R}_t^D , the expected return to lending. \bar{R}_t^D is also the most agents can expect to earn by leveraging their wealth in some market λ to buy assets. Entrepreneurs should thus use their endowment w to produce and earn the highest return. They must then choose whether to borrow in some market $\lambda \in [0, 1]$ to scale up their production, where we include $\lambda = 1$ to denote no borrowing.

Consider an entrepreneur with wealth $w > \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of \bar{R}_t^D , the lowest available interest rate on loans. If she borrowed in some market $\lambda < w$, she could borrow more than $1 - w$. But there is no benefit to this extra borrowing given her capacity. Moreover, the interest rate in this market would be the same or higher than \bar{R}_t^D . So there is no advantage to going to markets $\lambda < w$ over going to market $\lambda = w$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$, and she would face the same interest rate \bar{R}_t^D . This too offers

no benefit over going to market $\lambda = w$. The best this entrepreneur can do is go to market $\lambda = w$ to borrow $1 - w$, although she could also achieve the same payoff going to any market $\lambda \in [\Lambda_t^D, w]$.

Next, consider an entrepreneur with wealth $w \leq \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of $R_t^D(w)$. If she borrowed in some market $\lambda < w$, she would be able to borrow more than $1 - w$, but she has no use for this extra borrowing. Moreover, the interest rate in this market would be higher than $R_t^D(w)$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$. But she would face a lower interest rate. The question is whether it is worth reducing capacity to obtain a lower rate. Her payoff from borrowing in market $\lambda \in [w, \Lambda_t^D]$ would be $\frac{w}{\lambda} [1 + y^* - (1 - \lambda)(1 + R_t(\lambda))]$. Substituting in from (24), this is equal to

$$\frac{w}{\lambda} \left[1 + y^* - \frac{p_{t+1}^D + D}{p_t^D} + \frac{\lambda(1 + \bar{R}_t^D)}{1 - \pi} \right]$$

This payoff is decreasing in λ , so there is no advantage to borrowing in these markets instead of $\lambda = w$. Borrowing in any market $\lambda \in (\Lambda_t^D, 1)$ is dominated by borrowing in market $\lambda = \Lambda_t^D$, which we already argued was worse than borrowing in $\lambda = w$. So borrowing $1 - w$ in market $\lambda = w$ is uniquely optimal.

In any equilibrium where all markets are active, then, entrepreneurs with wealth $w \in [0, \Lambda_t^D)$ will borrow in market $\lambda = w$, while those with wealth $w \geq \Lambda_t^D$ would borrow in some market between Λ_t^D and w . This implies $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$ while $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. This indeterminacy is irrelevant for allocations or welfare, however, since in any such equilibrium we know agents with wealth $w \geq \Lambda_t^D$ borrow $1 - w$ at an interest rate of $1 + \bar{R}_t^D$. Just as before, we can ensure all markets are active by assuming entrepreneurs with wealth $w \geq \Lambda_t^D$ also borrow $1 - w$ in market $\lambda = w$.

Once again, any resources the young do not use to produce will be spent on the asset. This implies

$$p_t^D + 2\varphi e = (1 + \varphi) e \quad (25)$$

It follows that $p_t^D = (1 - \varphi) e$ for all t . This is the same price as when $d_t = d$. Although the price is the same, the expected return to buying the asset when $d_t = D$ is higher, with $1 + \bar{r}^D = \frac{(1 - \pi)D + \pi d}{(1 - \varphi)e}$.

As in Section 2, we managed to solve for the equilibrium price p_t^D and the interest rates $R_t^D(\lambda)$ for all markets λ without solving for the amounts people borrow to buy assets. We now solve for the amounts agents borrow to buy assets in each market, $f_t^a(\lambda)$. Recall that the expected return to lending $\bar{R}_t^D(\lambda) = \bar{R}_t^D$ for all active markets λ . Let $\alpha_t(\lambda) \equiv f_t^a(\lambda) / f_t(\lambda)$ denote the fraction of lending in any active market λ that is used to buy assets. Equating $\bar{R}_t^D(\lambda)$ with \bar{R}_t^D implies

$$(1 - \pi\alpha_t(\lambda)) R_t^D(\lambda) + \pi\alpha_t(\lambda) \left[\frac{d}{(1 - \varphi)e} - \Phi \right] = \bar{R}_t^D \quad (26)$$

For markets $\lambda < \Lambda_t^D$, the fact that $R_t^D(\lambda) > \bar{R}_t^D$ implies $\alpha_t(\lambda) > 0$. Some agents must borrow in these markets to speculate. Using the value of $R_t(\lambda)$ in (24), we can solve for $\alpha_t(\lambda)$ and then for $f_t^a(\lambda)$ using the fact that $f_t^p(\lambda) = 2\varphi e$. Since the interest rate on loans $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0, \Lambda_t^D)$, then

$\alpha_t(\lambda)$ and $f_t^a(\lambda)$ must be decreasing in λ for $\lambda < \Lambda_t^D$. There is more borrowing for speculation in markets with more leverage. This is not because leverage makes speculation more attractive, but because there must be just enough speculation in equilibrium to ensure the return to lending equals \bar{R}_t^D in all markets. There must be more borrowing for speculation in markets where lending to entrepreneurs is more profitable.

Earlier we established that agents do not borrow to buy assets in markets $\lambda \geq \Lambda_t^D$. Hence, $f_t^a(\lambda) = 0$ for $\lambda \in [\Lambda_t^D, 1)$, and so $f_t^a(\lambda)$ is uniquely determined for all $\lambda \in [0, 1)$ in any equilibrium in which all markets are active. We can also say something about who engages in speculation. In Section 2, who bought assets was indeterminate. This is still true for market $\lambda = 0$. But in markets $\lambda > 0$, borrowers must invest their own wealth to speculate. Entrepreneurs with $w < \Lambda_t^D$ invest all of their resources in production. So it must be savers and wealthy entrepreneurs who borrow to buy assets in markets $\lambda \in (0, \Lambda_t^D)$. But since entrepreneurs cannot all produce at capacity on their own, some savers must also lend in equilibrium.

We have now solved for the equilibrium price p_t^D , the interest rates on loans $R_t^D(\lambda)$, and the amounts $f_t^a(\lambda)$ and $f_t^p(\lambda)$ agents borrow to buy assets and produce for all λ . However, these are defined in terms of the expected return to saving \bar{R}_t^D , which we have yet to derive. To solve for \bar{R}_t^D , let us consider all savings in this economy. First, savings are used to finance production by entrepreneurs, which yields

$$\int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi e) dw$$

Second, savings are used to buy assets, directly or indirectly through loans. The expected earnings from these investments equal $(1 + \bar{r}_t^D) p_t^D$. From this, we must net out expected default costs. Let γ_t^D denote the fraction of spending on assets that is financed with some debt. These purchases will result in default if returns are low. Since default is proportional to the size of the borrower's project, expected default costs are equal to $\pi \gamma_t^D \Phi p_t^D = \pi \gamma_t^D \Phi (1 - \varphi) e$. Adding these up, these earnings must equal $(1 + \bar{R}_t^D) e$. Hence,

$$(1 + \bar{R}_t^D) e = [1 + \bar{r}^D - \pi \gamma_t^D \Phi] (1 - \varphi) e + \int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi e) dw \quad (27)$$

Finally, we need an additional equation to characterize γ_t^D . When the expected return to lending \bar{R}_t^D exceeds the expected return to buying the asset \bar{r}^D , only agents who borrow will buy the asset. In that case, $\gamma_t^D = 1$. When $\bar{R}_t^D = \bar{r}^D$, then γ_t^D would have to ensure that \bar{R}_t^D is indeed equal to \bar{r}^D . We can combine the two conditions into a single equation:

$$1 + \bar{R}_t^D = \max \left\{ 1 + \bar{r}^D, [1 + \bar{r}^D - \pi \Phi] (1 - \varphi) + \int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi) dw \right\} \quad (28)$$

It is easy to verify that when $\bar{R}_t^D > \bar{r}^D$, equations (27) and (28) imply $\gamma_t^D = 1$, and when $\bar{R}_t^D = \bar{r}^D$ we can find a unique value of γ_t^D that will equate the two. Since \bar{r}^D is time invariant, the solutions to these equations, \bar{R}^D and γ^D , are also time invariant. Given a value for \bar{R}^D , we can solve for the time invariant cutoff Λ^D as the smallest value of λ for which $R^D(\lambda) = \bar{R}^D$. This completes the characterization of an equilibrium when all markets are active, which yields the following analog to Proposition 2:

Proposition 7 *There exists an equilibrium in which all markets are active while $d_t = D$. In any such equilibrium, the asset price is given by*

$$p_t^D = (1 - \varphi)e \equiv p^D$$

and, in the limit as $\phi \rightarrow 0$, the interest rates on loans in different markets are given by

$$1 + R_t^D(\lambda) = \max \left\{ 1 + \bar{R}^D, \frac{1}{1-\lambda} \left[1 + \frac{D}{p^D} - \frac{\lambda(1+\bar{R}^D)}{1-\pi} \right] \right\}$$

where \bar{R}^D is the value that solves (27) and (28) together with γ^D . Borrowing for buying assets $f_t^a(\lambda)$ ensures $\bar{R}^D(\lambda) = \bar{R}^D$ for $\lambda \in [0, \Lambda^D)$ and $f_t^a(\lambda) = 0$ for $\lambda \in [\Lambda^D, 1)$. Borrowing for production is given by $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda^D)$, while for $\lambda \in [\Lambda^D, 1)$ must satisfy $\int_{\Lambda^D}^1 f_t^p(\lambda) d\lambda = (1 - (\Lambda^D)^2)\varphi e$.

As in Proposition 2, some agents blend in with entrepreneurs and borrow to buy assets. They do this in markets with high leverage, although not just in market $\lambda = 0$ where leverage is infinite. The high dividend regime still gives rise to credit booms and, if Φ isn't too large, bubbles. One difference from what we had before is that now all entrepreneurs produce at capacity, while before only an endogenous fraction of entrepreneurs did. As a result, an asset boom is no longer associated with misallocation: There is no production we could do instead of buying assets. However, borrowing to buy assets remains socially wasteful when $\Phi > 0$, and an intervention might still raise welfare by curbing excessive leverage.

So far, we have only considered equilibria in which all markets are active. But for any λ , we can always construct an equilibrium in which market λ is inactive by setting the interest rate on loans $R_t(\lambda)$ above y^* to ensure no agent would want to borrow in that market, and the expected return $\bar{R}_t(\lambda)$ to be arbitrarily low to ensure no one would want to lend in market λ . Such equilibria are essentially coordination failures where markets that could sustain trade are instead inactive. Inactivity in some markets will generally affect prices and interest rates in remaining active markets, and so characterizing equilibria with inactive markets would require us to solve again for interest rates, asset prices, and amounts borrowed. We will not try to characterize all such equilibria. However, we will now turn to studying interventions that shut down markets with low λ . This is equivalent to studying equilibria in which markets with low λ are inactive because of what agents believe rather than because they were shut down by fiat. The reason markets are inactive is irrelevant for how inactivity affects other markets. Given our interest in the effect of restricting markets that would otherwise trade, it seems natural to focus on equilibria in which markets are maximally active.

5.2 Leverage Restrictions

Proposition 7 implies speculators only borrow in markets with low λ . A natural way to intervene against speculation, then, is to shut down all markets λ below some floor $\underline{\lambda}$, or, alternatively, to cap the leverage agents can take on. Agents with wealth $w < \underline{\lambda}$ can only undertake projects of size at most $w/\underline{\lambda} < 1$. For simplicity we consider a permanent floor, although we could equally consider a floor only while $d_t = D$.

We restrict attention to equilibria in which all markets $\lambda \geq \underline{\lambda}$ are active. The equilibria in Proposition 7 is then a special case where $\underline{\lambda} = 0$. When $\underline{\lambda} > 0$, we can use the same arguments to show that interest

rates $R_t(\lambda)$ will be given by (24) when $d_t = D$. Entrepreneurs will therefore still want to invest all of their wealth w and borrow $1 - w$ to produce at full capacity. But entrepreneurs with $w < \underline{\lambda}$ can no longer do so. Since their profits are decreasing with λ for $\lambda > w$, these entrepreneurs will all flock to $\underline{\lambda}$ and produce at scale $w/\underline{\lambda}$. The total inputs entrepreneurs will use to produce is then

$$\begin{aligned} \int_{w=0}^{\underline{\lambda}} 2\varphi e \left(\frac{w}{\underline{\lambda}} \right) dw + \int_{w=\underline{\lambda}}^1 (2\varphi e) dw &= \frac{\varphi e}{\underline{\lambda}} w^2 \Big|_0^{\underline{\lambda}} + 2\varphi e (1 - \underline{\lambda}) \\ &= \underline{\lambda} \varphi e + (1 - \underline{\lambda}) 2\varphi e \end{aligned}$$

The amount that remains to spend on the asset is $(1 + \varphi)e$ minus the above, which pins down its price:

$$p_t^D = (1 - \varphi(1 - \underline{\lambda}))e \quad (29)$$

Increasing $\underline{\lambda}$ will lead to a higher asset price. Intuitively, leverage restrictions force poor entrepreneurs to operate at a smaller scale. Since savers want to save a fixed amount e regardless of $\underline{\lambda}$, the decline in production will release resources to buy assets, pushing p_t^D up. When $\underline{\lambda}$ is imposed permanently, the same logic implies $p_t^d = (1 - \varphi(1 - \underline{\lambda}))e$. The expected return on the asset when $d_t = D$ is therefore

$$1 + \bar{r}_t^D = \frac{(1 - \pi)(D + p_{t+1}^D) + \pi(d + p_{t+1}^d)}{p_t^D} = 1 + \frac{(1 - \pi)D + \pi d}{(1 - \varphi(1 - \underline{\lambda}))e}$$

Increasing $\underline{\lambda}$ thus reduces the expected return to buying the asset. It is hard to summarize the effects of increasing $\underline{\lambda}$ on the entire schedule of interest rates $R_t(\lambda)$, but we show in Appendix C that the expected return to lending \bar{R}^D declines with $\underline{\lambda}$. Intuitively, increasing $\underline{\lambda}$ depresses demand for credit, and so should lower interest rates. We can summarize the effects of raising $\underline{\lambda}$ as follows:

Proposition 8 *The asset price $p_t^D = (1 - \varphi(1 - \underline{\lambda}))e$ is increasing in $\underline{\lambda}$, while the expected returns on the asset $\bar{r}^D = \frac{(1 - \pi)D + \pi d}{(1 - \varphi(1 - \underline{\lambda}))e}$ and from lending \bar{R}^D are decreasing in $\underline{\lambda}$ for a permanent floor $\underline{\lambda}$.*

Note how this intervention compares with contractionary monetary policy in the previous section. Both policies reduce output. However, tighter monetary policy reduces the resources e_0 agents have access to today, while leverage restrictions reduce the amount entrepreneurs produce for next period by forcing poor entrepreneurs to operate below capacity. This difference implies that in our model, the two interventions have the opposite effect on asset prices and the return to savings. Nevertheless, tighter monetary policy and leverage restrictions might both discourage speculation. Even though leverage restrictions increase the asset price p^D , they also tend to reduce the share of assets purchased with debt γ^D . Indeed, setting $\underline{\lambda}$ above Λ^D will drive γ^D to 0 given that no agent will borrow to buy assets in markets $\lambda \geq \Lambda^D$. More generally, expected default costs are equal to $\pi\Phi\gamma^D p^D$. Whether increasing $\underline{\lambda}$ raises the deadweight loss from default depends on how increasing $\underline{\lambda}$ affects γ^D and p^D , respectively. Our next result shows that under certain conditions, increasing $\underline{\lambda}$ will increase p^D without changing γ^D . Specifically, this will be the case if the floor $\underline{\lambda}$ is already low and $\gamma^D = 1$, as well as if $\underline{\lambda}$ is already high enough to exceed Λ^D so that $\gamma^D = 0$. In these cases, raising $\underline{\lambda}$ will make agents worse off. But we also argue there exists an intermediate value of $\underline{\lambda}$ for which increasing $\underline{\lambda}$ will decrease γ^D enough to lower expected default costs $\pi\Phi\gamma^D p^D$. Increasing $\underline{\lambda}$ is thus ambiguous, and can in principle either increase or decrease welfare.

Proposition 9 *There exist cutoffs $\Lambda_0 \leq \Lambda_1 < 1$ in $[0, 1)$ such that*

1. *If $\underline{\lambda} < \Lambda_0$, increasing $\underline{\lambda}$ leaves $\gamma^D = 1$, increases expected default costs $\pi\gamma^D\Phi p^D$, and leaves fewer goods for cohorts to consume from date $t = 1$ on.*
2. *If $\underline{\lambda} \geq \Lambda_1$, $\gamma^D = 0$ and there is no default. Increasing $\underline{\lambda}$ then leaves fewer goods for cohorts to consume from date $t = 1$ on.*
3. *If $\Lambda_0 < \underline{\lambda} < \Lambda_1$, there exist values of $\underline{\lambda}$ at which increasing $\underline{\lambda}$ lowers γ^D and expected default costs $\pi\gamma^D\Phi p^D$. In this case, increasing $\underline{\lambda}$ while $d_t = D$ can be Pareto improving for large Φ .*

The fact that leverage restrictions can be counterproductive and increase speculation is a new result as far as we know. Stein (2013) argues leverage restrictions may be ineffective, but his point was that lenders borrowers can circumvent them, not that they might contribute to more speculation. The logic behind our result is that risk-shifting models require an additional investment activity to cross-subsidize speculation. If this other investment is particularly sensitive to leverage restrictions, restricting leverage may end up redirecting resources toward speculation. We anticipate that the same would hold true in risk-shifting models where speculators and less risky investors buy the same asset, as would be the case with housing. That is, if the demand for housing by liquidity constrained home buyers is particularly sensitive to leverage restrictions but the amount of funds available for mortgage lending is relatively inelastic with respect to interest rates, leverage restrictions could end up encouraging speculation on housing.⁶

While imposing leverage restrictions has ambiguous welfare effects, in our model a threat to restrict leverage in the future will unambiguously make things worse today. Recall that tighter monetary policy at date $t + 1$ will lower p_{t+1}^D , discouraging speculation at date t . By contrast, raising $\underline{\lambda}$ at date $t + 1$ will increase p_{t+1}^D . Regardless of how it affects γ_{t+1}^D , a higher p_{t+1}^D encourages speculation at date t . This contrast highlights how the two interventions affect asset prices and interest rates in opposite ways.

That said, we should be clear that while tighter leverage restrictions generally reduce demand for credit and lead to lower interest rates, the prediction of our model that this always leads to higher asset prices will not as naturally generalize. Suppose we let the wealth and productivity of entrepreneurs follow a general distribution $n(w, y)$. Entrepreneurs with low productivity would act like savers while entrepreneurs with high productivity would borrow to produce. An increase in $\underline{\lambda}$ that lowers the return to saving could induce some entrepreneurs who are on the margin to switch from lending out their wealth to borrowing in order to produce. If enough entrepreneurs switch from lending to producing, the fall in lending and the increase in

⁶While there are no direct analogs to our result, two recent papers similarly point out counterproductive aspects of central bank policies. Hachem and Song (2018) show that forcing banks to hold more liquidity may paradoxically lead to more interbank lending as large banks hold fewer reserves to hurt the small banks they compete with. Chen, Rhen, and Zha (2018) argue that contractionary monetary policy in China led to an increase in lending by shadow banks as a fall in deposits encouraged banks to lend more to shadow banks to avoid liquidity coverage requirements.

demand for borrowing to produce could leave fewer resources to spend on the asset, and its price will fall. We confirm numerically that there exist distributions $n(w, y)$ for which increasing $\underline{\lambda}$ reduces p_t^D .⁷

While increasing $\underline{\lambda}$ can in principle dampen asset prices, this will only be possible if there is risk-shifting. When $d_t = d$, increasing $\underline{\lambda}$ will raise p_t^d regardless of the distribution $n(w, y)$. To see this, recall that there is no default when $d_t = d$. In that case, the interest rate $R_t^d(\lambda)$ would equal the same R_t^d for all λ . This common rate R_t^d and the asset price p_t^d satisfy two equilibrium conditions similar to (3) and (4). First, since all resources must be used to produce or buy the asset, we have

$$\int_{R^d} \int_0^1 \min \left\{ 1, \frac{w}{\underline{\lambda}} \right\} n(w, y) dw dy + p_t^d = \int_0^\infty \int_0^1 wn(w, y) dw dy + e \quad (30)$$

This defines R_t^d as a function $\rho_{\underline{\lambda}}(p_t^d)$ of the price p_t^d which is increasing in p_t^d for a fixed $\underline{\lambda}$ and decreasing in $\underline{\lambda}$ for a fixed p_t^d . Second, the interest rate on loans must equal the return on the asset, and so

$$(1 + R_t^d) p_t^d = d + p_{t+1}^d \quad (31)$$

Substituting in $R_t^d = \rho_{\underline{\lambda}}(p_t^d)$ implies $p_{t+1}^d = \rho_{\underline{\lambda}}(p_t^d) - d$. Figure 4 illustrates the effect of increasing $\underline{\lambda}$ graphically. Since $\rho_{\underline{\lambda}}(p_t^d)$ is decreasing in $\underline{\lambda}$ for a fixed p_t , the curve that plots p_{t+1}^d as a function of p_t^d is lower for all $p_t^d > 0$, which implies a higher steady state p^d . Intuitively, increasing $\underline{\lambda}$ requires the interest rate on loans to fall so that credit markets clear even after demand for borrowing by poor entrepreneurs falls. Without risk-shifting, the interest rate on loans and the return on the asset are equal, so the latter must fall. A lower return on the asset implies a higher price. With risk-shifting, the interest rate on loans and the return on the asset can differ, so it will be possible for interest rates on loans to fall but the return on the asset to rise. As an aside, the fact that tighter leverage restrictions only reduce asset prices with risk-shifting suggests leverage restrictions could be used to detect risk-shifting empirically.

6 Conclusion

This paper analyzes policy in a risk-shifting model of asset prices. As in previous work on risk-shifting, we show our model can capture many observable features of asset and credit booms and busts. The general equilibrium framework we use allows us to go beyond this and analyze policy and welfare. We show that risk-shifting leads to misallocation and excessive leverage, creating a role for intervention. We then look at whether the leading policy proposals involving contractionary monetary policy and leverage restrictions can help mitigate these distortions. In our model, tighter monetary policy increases interest rates and lowers asset prices, which reduces excessive leverage but further inhibits investment that is already underfunded. Leverage restrictions have the opposite effect, lowering interest rates and, at often increasing

⁷Even without relying on a more general distribution $n(w, y)$, our results are in part due to our assumption that savers only like to consume when old, and so their saving is inelastic with respect to the interest rate. If we modified this, tighter leverage constraints that reduce the returns to savings could lead agents to save less and asset prices would fall.

asset prices. But they also discourage borrowing against assets. Both policies turn out to have ambiguous welfare implications. Whether a policy improves welfare depends on how it affects speculators vis-a-vis the productive activities that cross-subsidize them. It will also depend on how policy is implemented; credibly promising to tighten if a boom persists may improve welfare even when tightening immediately does not. Although for analytical tractability we analyze each policy separately, a consistent theme of our analysis is that an effective policy should disproportionately discourage speculation. Which policy is more effective depends on how policy is implemented, but also on how production and speculation respond to each policy, which in general requires an empirical answer. Finally, we find that when default costs are large, risk shifting can occur without giving rise to bubbles, something previous work has overlooked. This reveals that, given evidence of risk-shifting, policymakers contemplating intervening against asset booms might not need to determine if asset prices exceed fundamentals to justify their intervention.

We focus on risk shifting because asset booms often feature opaque assets where it is difficult for lenders to judge the risks from any given borrower. However, a large literature has analyzed bubbles without risk shifting. These models should not be viewed as competing explanations, since the mechanisms they consider are complementary to the risk-shifting we study. For example, there is a large literature showing bubbles can arise with fully rational agents because of dynamic inefficiency as in Galí (2014, 2017) or binding credit market frictions as in Martin and Ventura (2012), Hirano and Yanagawa (2017), and Miao and Wang (forthcoming). These models often focus on bubbles that burst stochastically. Since they feature risk, they can potentially give rise to risk-shifting. Bengui and Phan (2018) already showed how to combine risk-shifting and dynamic inefficiency. One can similarly combine risk-shifting and borrowing constraints by replacing our assumption that entrepreneurs have limited capacity with the assumption that their scale depends on how much they borrow. In that case, the distortions from risk-shifting we emphasize have to be balanced against the fact that overvalued assets may help relax borrowing constraints. A separate literature shows how disagreement about the risky returns on assets can give rise to bubbles, e.g. Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), Simsek (2013), and Barberis, Greenwood, Jin, and Shleifer (2018). Such differences in beliefs are certainly compatible with uncertainty about the risks lenders are exposed to. For example, we can allow savers in our model to hold different beliefs about the asset. Whether risk-shifting interacts with disagreement in interesting ways remains an open question.

Our model also suggests directions for future research on risk-shifting models of asset prices. For example, we assumed lenders suffer a cost Φ when their borrowers default. In practice, the main costs associated with the collapse of asset prices involve a decline in output due to the way agents respond when asset prices fall. To get at these channels would require introducing financial intermediaries or borrowing constraints for individual households. These may have important implications for what type of interventions are best during booms, since how interventions affect outcomes once asset prices collapse will likely matter for welfare. In terms of applications, we have described the analog between our setup and the housing market. However, cross-subsidization in the housing market works differently, since there both types of agents buy the same asset. By contrast, in our model the safe activity does not involve buying an asset. This raises the question of whether an intervention that shifts resources from illiquid home buyers to speculators still drives house

prices up as in our setting. It is also not obvious whether the policy implications we deduce in our model would hold in open economy settings. For example, we argued that a contractionary monetary policy raises interest rates and dampens asset prices. But if contractionary monetary policy leads to higher real rates that attract capital inflows, it is not clear whether asset prices will still fall. Extending our framework to deal with these issues, for example by considering an open economy version of our model along the lines of Galí and Monacelli (2005), is essential for figuring out its relevance and limitations for real world scenarios.

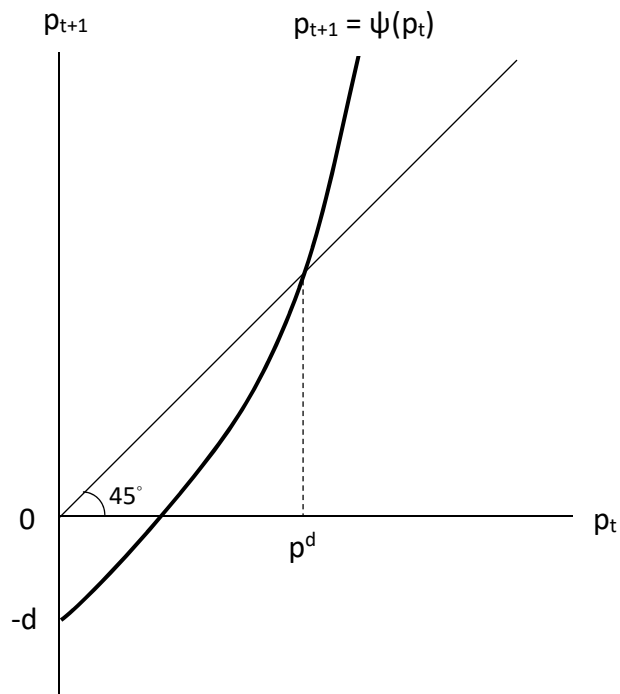


Figure 1: Determination of equilibrium price p^d with deterministic dividends

The value p^d denotes the steady state for the dynamical system $p_{t+1} = \psi(p_t)$. Any path which begins away from p^d leads either to a negative price or a price above e , neither of which can occur in equilibrium. Hence, the unique equilibrium is for the price to equal the steady state value p^d at all dates.

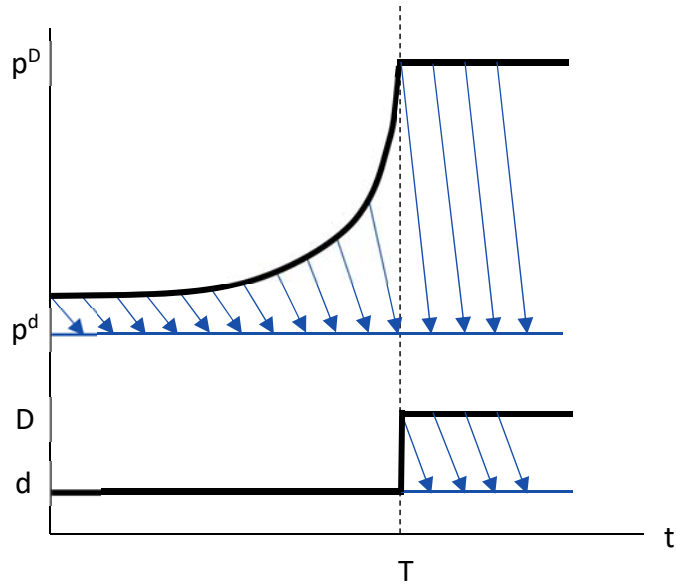


Figure 2: Equilibrium prices p_t^D with delayed dividend increase

The figure depicts the path of dividends and asset prices if dividends in the initial regime begin at d and switch to D if the regime remains unchanged through date T . Up through date T , prices in the initial regime follow an explosive path, even as dividends remain unchanged. From date T on, the price would remain constant p^D as long the regime remains unchanged. The blue arrows indicate the change in prices and dividends if the regime changes. A regime change permanently lowers the dividend to d and the asset price to p^d .

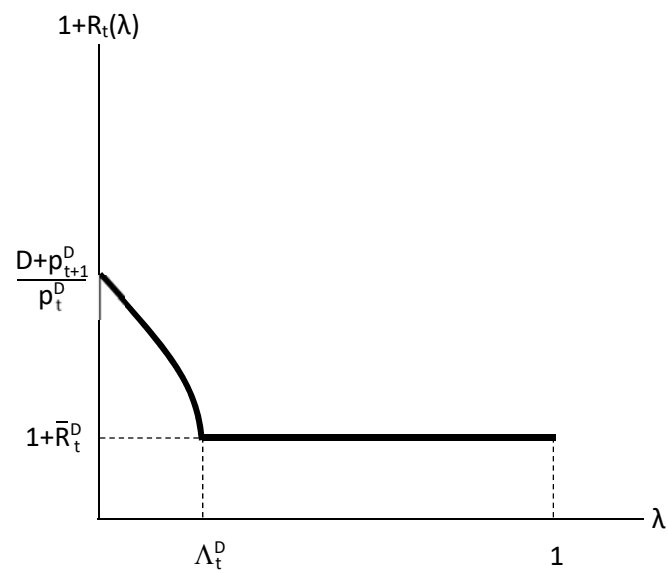


Figure 3: Interest rates as a function of share λ of investment that borrowers pay

The figure depicts the equilibrium schedule of interest rates across different markets. Interest rates are declining in the share λ of their projects that borrowers finance. For $\lambda < \Lambda_t^D$ the interest rate is falling in λ , and for $\lambda \geq \Lambda_t^D$ it is constant.

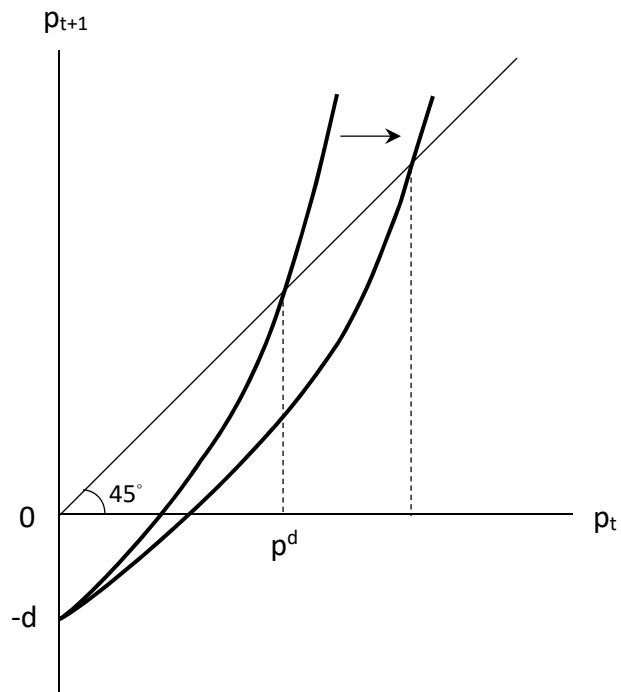


Figure 4: Effect of increasing floor $\underline{\lambda}$ with deterministic dividends

The figure depicts the dynamical system $p_{t+1} = \psi(p_t)$ for two values of $\underline{\lambda}$. The higher value represents the curve on the right, and is associated with a higher steady state price.

Appendix A: Proof of Proposition 1

Proof of Proposition 1: In the text, we showed there is a unique deterministic equilibrium. Here we allow for stochastic equilibrium paths for $\{p_t, R_t\}_{t=0}^{\infty}$ and confirm that the equilibrium is in fact deterministic.

First, note that for any date t , in equilibrium it must be the case that $0 < p_t \leq e$. If the price $p_t \leq 0$ there would be infinite demand for the asset given its dividend $d > 0$ and there is free disposal. But the supply of assets is finite, so this cannot be an equilibrium. At the same time, the most any cohort can spend to buy the assets is e . Let z_t denote the return to buying the asset, i.e., $z_t \equiv \frac{d+p_{t+1}}{p_t}$. This can be random if p_{t+1} is random. Let $G_t(z)$ denote the (possibly degenerate) distribution of the return z_t . Since $0 < p_t \leq e$ for all t , the maximum return z_t^{\max} is finite, since $z_t^{\max} = \frac{d+p_{t+1}^{\max}}{p_t} \leq \frac{d+e}{p_t} < \infty$, where p_{t+1}^{\max} is the maximum possible realization of the price at date $t+1$.

The equilibrium satisfies two conditions. First, as in (3), all resources will be used either to buy assets or to initiate production:

$$\int_{R_t}^{\infty} n(y) dy + p_t = e \quad (32)$$

This implies $R_t = \rho(p_t)$ where $\rho'(\cdot) > 0$. Second, the interest rate on loans R_t must satisfy

$$(1 + R_t) p_t = d + p_{t+1}^{\max} \quad (33)$$

If the interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, no agent would want to buy assets, which cannot be an equilibrium. If interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, agents could earn positive profits from borrowing, so demand for credit would be infinite. Substituting $R_t = \rho(p_t)$ into (33) implies

$$p_{t+1}^{\max} = (1 + \rho(p_t)) p_t - d$$

Suppose $p_t > p^d$. Consider the sequence $\{\tilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t . Formally, set $\tilde{p}_t = p_t$ and define

$$\tilde{p}_{\tau+1} = (1 + \rho(\tilde{p}_\tau)) \tilde{p}_\tau - d$$

Since $p_t > p^d$, the sequence \tilde{p}_t would shoot off to infinity and would exceed e in finite time. This means there is a state of the world in which the price exceeds e , which cannot be an equilibrium. So $p_t \leq p^d$.

Next, suppose $p_t < p^d$. Again, we can construct the sequence $\{\tilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t . That is, we set $\tilde{p}_t = p_t$ and then

$$\tilde{p}_{\tau+1} = (1 + \rho(\tilde{p}_\tau)) \tilde{p}_\tau - d$$

Since $p_t < p^d$, the sequence \tilde{p}_t would turn negative. Hence, there is a state of the world in which the price is negative, which cannot be an equilibrium. The distribution of the price at date t is degenerate with full support at p^d . From (32), $R_t = \rho(p_t)$ is uniquely determined as well. ■

Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup as in our discussion in Section 4. We set up the model and derive the results that underlie Propositions 4 and 5 in the text.

B.1 Agent Types and Endowments

Our approach largely follows Galí (2014) in how we incorporate production, nominal price rigidity, and monetary policy into an overlapping generations economy with assets. As in our benchmark model, agents live two periods and care only about consumption when old. Each cohort still consists of two types – savers who are endowed with resources but cannot produce intertemporally and entrepreneurs endowed with no resources who can convert goods at date t into goods at date $t + 1$. We continue to model entrepreneurs as in the benchmark model, but we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each who must choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.⁸ Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date t convert labor into goods at date t . Entrepreneurs then convert the goods producers created at date t into goods at date $t + 1$. Producers operate within the period; entrepreneurs operate across periods.

B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor to home and market production. Home production yields the same good as the market, but using a technology $h(\ell)$ that is concave in the amount of labor ℓ allocated to home production. We assume $h'(0) = 1$ and $h'(1) = 0$ for reasons that will become clear below.

Workers who sell their labor on the market earn a wage W_t per unit labor. Their labor services are hired by producers, whom we index by $i \in [0, 1]$. Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer i hires n_{it} units of labor, she will produce $x_{it} = n_{it}$ units of intermediate good i . The different intermediate goods can then be combined to form final

⁸This setup borrows from Adam (2003) rather than Galí (2014). The latter assumes agents are homogeneous, selling labor when young and hiring labor when old. We want income to only accrue to the young as in our benchmark model.

consumption goods according to a constant elasticity of substitution (CES) production function available to all agents. That is, given x_{it} of each $i \in [0, 1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (34)$$

Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i . At these prices, the x_{it} that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields

$$x_{it} = X_t \left(\frac{P_{it}}{P_t} \right)^{-\frac{1}{\sigma}} \quad (35)$$

If we set $X_t = 1$, we can compute the price of the cost of the optimal bundle of intermediate goods $x_{it} = \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma}$ needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (36)$$

Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (35) and wage W_t . To allow producers to move either before or after the monetary authority, we condition producer i 's choice on their information Ω_{it} when choosing their price. Each producer will set P_{it} to solve

$$\max_{P_{it}} E \left[(P_{it} - W_t) X_t \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is then

$$P_{it} = \frac{E[W_t X_t | \Omega_{it}]}{(1-\sigma) E[X_t | \Omega_{it}]} \quad (37)$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0, 1]$. The output of consumption goods is thus

$$X_t = \left(\int_0^1 n_t^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive $(W_t/P_t)n_t$ of these goods and producers get the remaining $(1 - W_t/P_t)n_t$. Workers also produce goods at home. Their income is thus $(W_t/P_t)n_t + h(1 - n_t)$, which is maximized at

$$h'(1 - n_t) = W_t/P_t \quad (38)$$

By contrast, the total resources available to young agents is $e_t = n_t + h(1 - n_t)$, which is maximized at

$$h'(1 - n_t) = 1$$

Our assumption that $h'(0) = 1$ implies total resources are maximized when $n_t = 0$ and all goods are produced in the market, and $e_t = n_t + h(1 - n_t)$ is increasing in n_t for all $n_t \in [0, 1]$.

B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings $e_t = n_t + h(1 - n_t)$. As in the benchmark model, they can buy assets and make loans. Without money, this specification would be equivalent to our benchmark model, the only difference being that the income of savers e_t which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y) dy + p_t = e_t$$

where p_t is the real price of the asset and R_t is the real interest rate on loans. The interest rate R_t must still ensure agents cannot earn profits by borrowing and buying assets. When $d_t = d$, this requires

$$(1 + R_t^d) p_t^d = d + p_{t+1}^d$$

and when $d_t = D$, this requires

$$(1 + R_t^D) p_t^D = D + p_{t+1}^D$$

We can then use R_t and p_t to solve for the expected return on loans:

$$\bar{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max \left\{ \bar{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t} (\bar{r}_t^D - \pi\Phi) \right\} & \text{if } d_t = D \end{cases} \quad (39)$$

where \bar{r}_t^D is the expected real return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage W_t/P_t will be constant over time. Employment n_t and total earnings of all savers $e_t = n_t + h(1 - n_t)$ will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and P_t and W_t denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. The monetary authority announces a nominal interest rate i_t at each date t . The monetary authority commits to pay this rate at date $t + 1$ to those who lend to it (with money it can always issue), and will charge i_t to those who borrow from it with full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t + 1$. Since agents always lend to entrepreneurs, the expected return on savings will equal \bar{R}_t , the expected return on loans. This implies

$$1 + i_t = (1 + \bar{R}_t) \Pi_t \quad (40)$$

When the monetary authority changes the nominal interest rate i_t , either inflation Π_t or the expected return $1 + \bar{R}_t$ or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

B.4 Defining an Equilibrium

Given a path of nominal interest rates $\{1 + i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

$$\begin{aligned}
\text{(i) Optimal pricing:} & \quad P_t = \frac{E[W_t X_t | \Omega_t]}{(1 - \sigma) E[X_t | \Omega_t]} \\
\text{(ii) Optimal labor supply:} & \quad h'(1 - n_t) = W_t / P_t \\
\text{(iii) Optimal saving:} & \quad \int_{R_t}^{\infty} n(y) dy + p_t = e_t \\
\text{(iv) Credit market clearing:} & \quad 1 + R_t = \begin{cases} \frac{D + p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d + p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases} \\
\text{(v) Money market clearing:} & \quad \Pi_t = \frac{1 + i_t}{1 + \bar{R}_t}
\end{aligned}$$

where the expected return on loans \bar{R}_t in the last condition is given by (39).

B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices P_{it} after observing the wage W_t . This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output X_t . Hence, their information set $\Omega_t = \{W_t, X_t\}$. It follows that $E[W_t X_t | \Omega_t] = W_t X_t$ and $E[X_t | \Omega_t] = X_t$. The optimal pricing rule (i) then implies

$$P_t = \frac{W_t}{1 - \sigma}$$

The real wage is thus constant and equal to $1 - \sigma$. Substituting this into (ii) yields

$$h'(1 - n_t) = 1 - \sigma \tag{41}$$

Since $h(\cdot)$ is concave, n_t is equal to some constant n^* for all t . It follows that $e_t = n^* + h(1 - n^*)$ is also constant for all t . We can then use (iii) and (iv) to solve for p_t and R_t as in the benchmark model, and then use (39) to compute \bar{R}_t . Finally, given \bar{R}_t we can use the implied Π_t from (v) to derive $\{P_t\}_{t=1}^{\infty}$ for any initial value for P_0 . The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage $W_t = (1 - \sigma) P_t$.

B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good P_{it} before the monetary authority moves. That is, producers set prices, the monetary authority sets $1 + i_t$, and then producers hire workers at a nominal wage W_t . This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage W_t , and so $\Omega_t = \{W_t, X_t\}$ and $W_t/P_t = 1 - \sigma$ as before.

Next, suppose monetary policy is contingent on some random variable, i.e., $i_t = i(\xi_t)$ where $\{\xi_t\}_{t=0}^\infty$ is a sequence of random variables. For simplicity, consider the case where ξ_t is only random at $t = 0$, i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1 - \chi \end{cases}$$

ξ_t is deterministic for $t = 1, 2, \dots$

From date $t = 1$ on, we know from the optimal price-setting condition (i) that $W_t/P_t = 1 - \sigma$. It then follows that $n_t = n^*$ and $e_t = e^* \equiv n^* + h(1 - n^*)$ for all $t \geq 1$, and we can determine p_t , R_t , and \bar{R}_t for $t \geq 1$ just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as for a given realization of ξ_0 . Assume wlog that $i_0^H > i_0^L$. The optimal price setting condition (1) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1 - \chi) n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1 - \chi) n_0^L} = 1 - \sigma \quad (42)$$

That is, the output-weighted average real wage over the two values of ξ is equal to $1 - \sigma$. Optimal labor supply (ii) then implies

$$\begin{aligned} h'(1 - n_0^H) &= \min \left\{ \frac{W_0^H}{P_0}, 1 \right\} \\ h'(1 - n_0^L) &= \min \left\{ \frac{W_0^L}{P_0}, 1 \right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we choose the real wage when $\xi = H$ to be this parameter. The three equations above yield values for W_0^L/P_0 , n_0^H , and n_0^L given W_0^H/P_0 . From these, we can deduce earnings $e_0^\xi = n_0^\xi + h(1 - n_0^\xi)$ for each $\xi \in \{H, L\}$. We can then use (iii) and (iv) to derive p_0^ξ and R_0^ξ by solving

$$\int_{R_0^\xi}^\infty n(y) dy + p_0^\xi = e_0^\xi \quad (43)$$

$$(1 + R_0^\xi) p_0^\xi = D + p^D \quad (44)$$

and then compute the expected return on loans \bar{R}_0^ξ using (39), and, via (v), the inflation rate Π_0^ξ for each $\xi \in \{H, L\}$. As before, the price level P_0 is indeterminate. Optimal pricing only restricts the average real wage across states but not the real wage for each realization of ξ_0 , introducing an indeterminacy. The equilibrium real wage can exceed $1 - \sigma$ for one realization of ξ_0 if it falls below $1 - \sigma$ for the other realization.

There case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case, $W_0^H/P_0 = W_0^L/P_0 = 1 - \sigma$. But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal to

coordinate real activity rather but does not directly affect it, there are equilibria in which $W_0^H > W_0^L$ as well as equilibria in which $W_0^H < W_0^L$.⁹ Since higher nominal interest rates seem to be contractionary in practice, we focus on equilibria in which $W_0^H/P_0 < 1 - \sigma < W_0^L/P_0$, i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from condition (ii) we know that a higher nominal interest rate will be associated with lower employment ($n_0^H < n^* < n_0^L$) and hence lower earnings ($e_0^H < e^* < e_0^L$). From (43), we can infer that $R_0^\xi = \rho^\xi(p_0^\xi)$ where $\rho^H(x) > \rho^L(x)$ for the same value x . As is clear from Figure 1, this implies a higher nominal interest rate will be associated with a lower real asset price ($p_0^H < p^D < p_0^L$). This also implies a higher real interest rate on loans ($R_0^H > R^D > R_0^L$). The real expected return to buying assets will also be higher ($\bar{r}_0^H > \bar{r}^D > \bar{r}_0^L$). However, whether the real expected return to lending \bar{R}_0^H will be higher is ambiguous. (39) implies \bar{R}_0^ξ is either equal to \bar{r}_0^ξ or to a weighted average of R_0^ξ and \bar{r}_0^ξ . In the latter case, although both terms are higher when $\xi = H$ the weight on \bar{r}_0^ξ , which is p_0^ξ/e_0^ξ , can be higher or lower for $\xi = H$. These results are summarized in Proposition 4 in the paper.

B.7 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case, ξ_0 and ξ_t for $t \geq 2$ are deterministic, while $\xi_1 = d_1 \in \{d, D\}$. That is, we assume producers set prices each period before d_t is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed ξ_0 was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1 - \sigma < W_1^d/P_1.$$

This implies $n_1^D < n^* < n_1^d$ and so $e_1^D < e < e_1^d$. In other words, if dividends fall and the boom ends, monetary policy must be expansionary. By the same logic as above, such a policy would imply $p_1^D < p^D$ and $p_1^d > p^d$, as well as $R_1^D < R^D$ and $R_1^d > R^d$. Turning back to date 0, conditions (iii) and (iv) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n(y) dy + p_0^D &= e \\ (1 + R_0^D) p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to p_1^D reveals that $p_0^D < p^D$ and $R_0^D < R^D$. That is, while contractionary monetary policy at date 0 dampens p_0^D but raises R_0^D at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both p_0^D and R_0^D at date 0. These results are summarized in Proposition 5 in the paper.

⁹One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to take actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

Appendix C: Macroprudential Regulation

In this appendix, we define an equilibrium for an economy with multiple markets as in Section 5. We then show that for an equilibrium in which all markets are active, various aspects of the equilibrium are uniquely determined. We then discuss some comparative static results with respect to the set of active markets.

C.1 Defining an Equilibrium

We begin with some notation. Let p_t denote the price of the asset at date t . Given asset prices, we can define the return to buying the asset at date t as

$$z_t \equiv \frac{d_{t+1} + p_{t+1}}{p_t}$$

The return z_t can be random both because d_{t+1} might be uncertain (if $d_t = D$) and because p_{t+1} might in principle be stochastic. Let $G_t(z)$ denote the (possibly degenerate) cumulative distribution of the return z_t , i.e., $G(z) \equiv \Pr(z_t \leq z)$. Let $1 + r_t^{\max}$ denote the maximum possible return on the asset. As discussed in the text, $1 + r_t^{\max}$ is finite, since $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e}$. We will use \bar{r}_t to denote the expected return to buying the asset at date t , i.e.,

$$1 + \bar{r}_t \equiv \int_0^{1+r_t^{\max}} z_t dG_t(z)$$

We now define variables for the different markets $\lambda \in [0, 1)$ agents can borrow in. Let $R_t(\lambda)$ denote the interest rate on loans in market λ , so an agent who agrees to pay a share λ of the project she undertakes will promise to pay back $1 + R_t(\lambda)$ for each unit she borrows. Since agents may default, let $\bar{R}_t(\lambda)$ denote what lenders expect to earn from lending in market λ given the possibility of default. Finally, we represent borrowing in markets with density functions $f_t^a(\lambda)$ and $f_t^p(\lambda)$ for all $\lambda \in [0, 1)$ such that the total amount of resources borrowed to buy assets and produce are given by $\int_A f_t^a(\lambda) d\lambda$ and $\int_A f_t^p(\lambda) d\lambda$, respectively. Let $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote the density of borrowing for any purpose in market λ .

Representing the quantities agents borrow in each market as a density function ignores the possibility that there may be equilibria in which agents borrow a positive mass of resources in certain markets. More generally, we can allow for a set $\Delta \subset [0, 1)$ with countably many elements such that each market $\lambda \in \Delta$ is associated with a positive mass of borrowing $m_t^x(\lambda) > 0$. The amount borrowed in any market $\lambda \in [0, 1) \setminus \Delta$ can still be represented with a density function. Heuristically, we can appeal to the Dirac-delta construction and represent the amount borrowed in any market as if it were a density. That is, for any $\lambda \in \Delta$, we set the density $f_t^x(\lambda) = m_t^x(\lambda) \delta_\lambda(\lambda)$, where $\delta_\lambda(q)$ is the Dirac-delta function defined so that $\delta_\lambda(q) = 0$ for $q \neq \lambda$ and $\int_0^1 \delta_\lambda(q) dq = 1$. This convention treats markets $\lambda \in \Delta$ as essentially having an infinite density. We will refer to a market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$ or if $\lambda \in \Delta$.

Given these preliminaries, we define an equilibrium as a path $\{p_t, f_t^p(\lambda), f_t^a(\lambda), R_t(\lambda), \bar{R}_t(\lambda)\}_{t=0}^\infty$ that satisfies conditions (45)-(50) below to ensure that all markets clear when agents are optimizing.

Our first three conditions stipulate that agents act optimally. We begin with lenders. Optimality requires that agents will only invest their wealth where the expected return is highest. Let \bar{R}_t denote the maximal expected return to lending in any market λ , i.e.,

$$\bar{R}_t \equiv \sup_{\lambda \in [0,1]} \bar{R}_t(\lambda)$$

Optimal lending requires that agents lend in market λ' only if they expect to earn \bar{R}_t and if this rate exceeds the expected return to buying the asset, i.e.,

$$f_t(\lambda') > 0 \text{ only if } \bar{R}_t(\lambda') = \bar{R}_t \text{ and } \bar{R}_t \geq \bar{r}_t \quad (45)$$

Next, entrepreneurs must act optimally. We first argue this means they should use their endowment to produce. Recall entrepreneurs have productivity y^* where $y^* > r_t^{\max} \geq \bar{r}_t$ from (20), so producing is better than buying assets. But y^* must also exceed the expected return to lending \bar{R}_t . For suppose \bar{R}_t were higher than y^* . Since $y^* > r_t^{\max}$, then \bar{R}_t must also exceed r_t^{\max} . In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least \bar{R}_t . Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since production offers the highest return, entrepreneurs should use their endowment w to produce.

Since entrepreneurs can leverage their endowment to produce at a larger capacity, we also need to characterize their borrowing. If they borrow in market λ where $\lambda < w$, they can borrow enough to reach full capacity. Optimality requires that there will be borrowing to produce in market λ' only if some entrepreneur finds it optimal to borrow in that market from all $\lambda \in [0, 1]$, including $\lambda = 1$ for no borrowing. This implies

$$f_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg \max_{\lambda \in [0,1]} \left\{ \begin{array}{ll} [1 + y - (1 - w)(1 + R_t(\lambda))] & \text{if } \lambda \leq w \\ \frac{w}{\lambda} [1 + y - (1 - \lambda)(1 + R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } w \quad (46)$$

Third, agents who borrow to buy assets must act optimally. They will agree to borrow in market $\lambda \in [0, 1)$ to buy assets only if doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1 - \lambda)$$

The expected profits from borrowing in market λ to buy one consumption unit's worth of assets is

$$\int_{x_t(\lambda)}^{\infty} (z_t - x_t(\lambda)) dG(z_t) \quad (47)$$

Agents will borrow in market λ to buy assets only if (47) equal $(1 + \bar{R}_t)\lambda$, the return on what they must spend on assets. If (47) were lower than $(1 + \bar{R}_t)\lambda$, no agent would borrow to buy assets. If (47) were higher than $(1 + \bar{R}_t)\lambda$, then no one would ever lend given they can borrow in market λ' , and so $f_t(\lambda') = 0$. But this contradicts the fact that $f_t^a(\lambda) > 0$. Optimality implies

$$f_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda')) dG(z_t) = (1 + \bar{R}_t)\lambda' \quad (48)$$

Fourth, savers will not waste any resources. Since entrepreneurs use their endowment to produce, all the resources spent to buy the asset must come from savers. This implies that e must be either lent to entrepreneurs to produce or be spent on assets:

$$\int_0^1 f_t^p(\lambda) d\lambda + p_t = e \quad (49)$$

Finally, we turn to equilibrium beliefs. In any active market λ' , lenders must expect the return on lending $\bar{R}_t(\lambda')$ to conform with the actual fraction of borrowers who borrow in market λ' with the intent to produce and to buy assets, respectively. That is,

$$\bar{R}_t(\lambda') = \frac{f_t^p(\lambda')}{f_t(\lambda')} R_t(\lambda') + \frac{f_t^a(\lambda')}{f_t(\lambda')} E_t \max \left\{ R_t(\lambda'), \frac{d_{t+1} + p_{t+1}}{p_t} - 1 \right\} \text{ if } f_t(\lambda') > 0 \quad (50)$$

In a market $\lambda \in \Delta$ with a positive mass of borrowing, the expression $\frac{f_t^x(\lambda')}{f_t(\lambda')}$ will be replaced by $\frac{m_t^x(\lambda)}{m_t(\lambda)}$. Condition (50) does not impose any restrictions on expectations in inactive markets where $f_t(\lambda') = 0$.

C.2 Solving for Equilibrium

We now proceed to solve for an equilibrium. As in the text, we restrict attention to equilibria in which all markets $\lambda \in [0, 1)$ are active. Such equilibria are natural given we focus on the effects of interventions to shut down markets. Our first result characterizes the schedule of interest rates in such an equilibrium.

Proposition C1: *In an equilibrium where all markets are active, there exists a value $\Lambda_t \in [0, 1]$ such that the equilibrium interest rate schedule will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\tilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \Lambda_t) \\ 1 + \bar{R}_t & \text{if } \lambda \in [\Lambda_t, 1) \end{cases} \quad (51)$$

where $\tilde{x}_t(\lambda)$ is the value of x that solves

$$\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z) = (1 + \bar{R}_t) \lambda \quad (52)$$

The schedule of interest rates $R_t(\lambda)$ is a decreasing and continuous function of λ for $\lambda \in [0, \Lambda_t]$.

Proof of Proposition C1: Our proof relies proceeds as two lemmas.

Lemma C1: In an equilibrium where all markets are active, $1 + R_t(\lambda) = \max \left\{ \frac{\tilde{x}_t(\lambda)}{1-\lambda}, 1 + \bar{R}_t \right\}$, where $\tilde{x}_t(\lambda)$ equals the x that solves (52) and \bar{R}_t is the expected return to lending in any market λ .

Proof of Lemma C1: Recall we defined $x_t(\lambda) \equiv (1 + R_t(\lambda))(1 - \lambda)$ as the equilibrium debt obligation for an agent who invests one unit of resources in assets. As we argued above, for all λ we have

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda)) dG_t(z) \leq (1 + \bar{R}_t) \lambda \quad (53)$$

since otherwise agents would refuse to lend, which is incompatible with $f_t(\lambda) > 0$ for all $\lambda \in [0, 1)$. The expression $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$ is strictly decreasing in x . It also tends to $+\infty$ as $x \rightarrow -\infty$ and to 0 as $x \rightarrow 1+r_t^{\max}$. Hence, for any $\lambda \in [0, 1)$ and any $\bar{R}_t \geq 0$, there exists a unique $x \in (-\infty, 1+r_t^{\max}]$ for which

$$\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z) = (1 + \bar{R}_t) \lambda \quad (54)$$

Denote $\tilde{x}_t(\lambda)$ as the unique solution to equation (54). By contrast, $x_t(\lambda)$ refers to the value of $(1 + R_t(\lambda))(1 - \lambda)$ evaluated at the equilibrium interest rate $R_t(\lambda)$.

For any $\lambda' \in [0, 1)$ in which (53) holds with equality, we have $\tilde{x}_t(\lambda') = x_t(\lambda')$, and so

$$1 + R_t(\lambda') = \frac{\tilde{x}_t(\lambda')}{1 - \lambda'}$$

For any remaining values of $\lambda' \in [0, 1)$, condition (53) holds as a strict inequality. This means borrowing in market λ' and buying assets yields a lower payoff than lending out the resources needed to borrow in market λ' . Hence, no agent will borrow in market λ' to buy assets, implying $f_t^a(\lambda') = 0$. In an equilibrium where all markets are active, $f_t^p(\lambda') > 0$. From (45) we know that $\bar{R}_t(\lambda') = \bar{R}_t$, and from (50) we know that since $f_t^a(\lambda') = 0$ then $\bar{R}_t(\lambda') = R_t(\lambda')$. Combining these implies $R_t(\lambda') = \bar{R}_t$.

Hence, in an equilibrium where all markets are active, we must have either $R_t(\lambda) = \bar{R}_t$ or $R_t(\lambda) = \frac{\tilde{x}_t(\lambda)}{1-\lambda}$ for all $\lambda \in [0, 1)$. To further show that $1 + R_t(\lambda) = \max\left\{1 + \bar{R}_t, \frac{\tilde{x}_t(\lambda)}{1-\lambda}\right\}$, consider a value of λ for which $\frac{\tilde{x}_t(\lambda)}{1-\lambda} > 1 + \bar{R}_t$, i.e., for which $\tilde{x}_t(\lambda) > x_t(\lambda)$. Since $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$ is decreasing in x , this means

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z-x_t(\lambda)) dG_t(z) > \int_{z=\tilde{x}(\lambda)}^{1+r_t^{\max}} (z-\tilde{x}(\lambda)) dG_t(z) = (1 + \bar{R}_t) \lambda$$

Since in equilibrium we must satisfy (53), it follows that in this case we have $1 + R_t(\lambda) = \frac{\tilde{x}_t(\lambda)}{1-\lambda}$.

Next, consider a value of λ for which $\frac{\tilde{x}_t(\lambda)}{1-\lambda} < 1 + \bar{R}_t$, i.e., for which $\tilde{x}_t(\lambda) < x_t(\lambda)$. Then we would have

$$(1 + \bar{R}_t) \lambda = \int_{z=\tilde{x}(\lambda)}^{1+r_t^{\max}} (z-\tilde{x}(\lambda)) dG_t(z) > \int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z-x_t(\lambda)) dG_t(z)$$

In this case, (53) can only hold as a strict inequality. But we already know that in this case $R_t(\lambda) = \bar{R}_t$. This establishes the lemma. ■

Our next lemma establishes that $\frac{\tilde{x}_t(\lambda)}{1-\lambda}$ is a weakly decreasing and continuous function of λ . Combined with Lemma C1, this implies there exists a cutoff Λ_t such that $R_t(\lambda) = \bar{R}_t$ for $\lambda \geq \Lambda_t$.

Lemma C2: In any equilibrium where all markets are active, $\frac{\tilde{x}_t(\lambda)}{1-\lambda}$ is nonincreasing and continuous in λ .

Proof of Lemma C2: The function $\tilde{x}_t(\lambda)$ corresponds to the value of x which solves (52). Although the distribution $G_t(z)$ can contain mass points, the integral $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$ is still continuous in x .

This implies $\tilde{x}_t(\lambda)$ is a continuous function of λ . However, $\tilde{x}_t(\lambda)$ may exhibit kinks, meaning its directional derivatives need not be equal at all values. To show that $\tilde{x}_t(\lambda)$ is decreasing, it will suffice to show that all of its directional derivatives are nonpositive for all $\lambda \in [0, 1)$. Totally differentiating (52) implies

$$\frac{d\tilde{x}_t(\lambda)}{d\lambda} = -\frac{1 + \bar{R}_t}{\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

For any λ where $\tilde{x}_t(\lambda)$ is a mass point of $G_t(z)$, $\lim_{\lambda' \rightarrow \lambda^+} \int_{\tilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda' \rightarrow \lambda^-} \int_{\tilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z)$. Nevertheless, both $\lim_{\lambda' \rightarrow \lambda^+} \frac{d\tilde{x}_t(\lambda')}{d\lambda'}$ and $\lim_{\lambda' \rightarrow \lambda^-} \frac{d\tilde{x}_t(\lambda')}{d\lambda'}$ are negative, so $\tilde{x}_t(\lambda)$ is strictly decreasing in λ .

Next, define $\tilde{R}_t(\lambda) \equiv \frac{\tilde{x}_t(\lambda)}{1-\lambda} - 1$. The function $\tilde{R}_t(\lambda)$ is also continuous in λ with possible kink-points. Differentiating the equation $\tilde{x}_t(\lambda) = (1-\lambda)(1 + \tilde{R}_t(\lambda))$ implies

$$\frac{d\tilde{x}_t(\lambda)}{d\lambda} = -(1 + \tilde{R}_t(\lambda)) + (1-\lambda) \frac{d\tilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for $\frac{d\tilde{x}_t(\lambda)}{d\lambda}$ above yields

$$\begin{aligned} \frac{d\tilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{1-\lambda} \left[1 + \tilde{R}_t(\lambda) + \frac{d\tilde{x}_t(\lambda)}{d\lambda} \right] \\ &= \frac{1}{1-\lambda} \left[1 + \tilde{R}_t(\lambda) - \frac{1 + \bar{R}_t}{\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)} \right] \\ &= \frac{1}{(1-\lambda) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)} \left[(1 + \tilde{R}_t(\lambda)) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - (1 + \bar{R}_t) \right] \end{aligned} \quad (55)$$

Once again, $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$ is discontinuous at λ where $\tilde{x}_t(\lambda)$ is a mass point of $G_t(z)$.

To evaluate the sign of $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$, we must consider two cases. First, suppose $\tilde{R}_t(\lambda) < \bar{R}_t$. Then

$$\begin{aligned} (1 + \tilde{R}_t(\lambda)) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) &< (1 + \bar{R}_t) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \\ &\leq 1 + \bar{R}_t \end{aligned}$$

In that case, we have $\frac{d\tilde{R}_t(\lambda)}{d\lambda} < 0$ from (55) regardless of the direction we take the derivative. Next, suppose $\tilde{R}_t(\lambda) \geq \bar{R}_t$. From Lemma C1, in this case (53) holds with equality. Rearranging this equation, we get

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z - (1 + \tilde{R}_t(\lambda)) \right] dG_t(z) = \lambda \left[(1 + \bar{R}_t) - \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1 + \tilde{R}_t(\lambda)) dG_t(z) \right]$$

We can establish that $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$ in (55) is nonnegative for $\lambda > 0$ if we can show that

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z - (1 + \tilde{R}_t(\lambda)) \right] dG_t(z) \geq 0$$

Towards this, observe that the expected profits from borrowing in market λ to buy assets are given by

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z)$$

Since these are equal to $(1+\bar{R}_t)\lambda$ when $\tilde{R}_t(\lambda) \geq \bar{R}_t$, we have

$$\begin{aligned} (1+\bar{R}_t)\lambda &= \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z) \\ &\leq \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_0^{1+r_t^{\max}} \lambda z_t dG(z) \\ &= \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + (1+\bar{r}_t^D)\lambda \end{aligned} \quad (56)$$

But in an equilibrium where all markets are active, we must have $\bar{R}_t^D \geq \bar{r}_t^D$. This implies

$$0 \leq (\bar{R}_t - \bar{r}_t^D)\lambda \leq (1-\lambda) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1+R_t)) dG(z)$$

This confirms $\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1+R_t)) dG(z) \geq 0$. All directional derivatives $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$ are nonnegative. ■

From Lemmas C1 and C2, set Λ_t to be either 1 or the minimum value in $[0, 1]$ for which $R_t(\lambda) = \bar{R}_t$. It follows that $R_t(\lambda) > \bar{R}_t$ for $\lambda < \Lambda_t$ and $R_t(\lambda) = \bar{R}_t$ for all $\lambda \geq \Lambda_t$. This establishes the proposition. ■

We can use the schedule of interest rates in Proposition C1 to determine how much entrepreneurs should produce and in which markets to borrow if they do.

Proposition C2: *In an equilibrium where all markets are active, entrepreneurs with wealth w will borrow $1-w$ units to produce, in a market with an interest rate equal to $R_t(w)$.*

Proof of Proposition C2: Consider an entrepreneur with wealth w . If she borrows in a market λ where $\lambda \leq w$, she can produce at full capacity and would only need to put down $\lambda \left(\frac{1-w}{1-\lambda} \right)$ resources to borrow $1-w$ to reach full capacity. This would earn her an expected profit of

$$1 + y^* - (1 + R_t(\lambda))(1 - w)$$

This value is maximized by choosing λ to minimize $R_t(\lambda)$. From Proposition C1, we know $R_t(\lambda)$ is weakly decreasing in λ and is therefore maximized at $\lambda = w$.

Next, suppose she borrows in a market λ where $\lambda > w$. In that case, she could not produce at full capacity. Since $y^* > r_t^{\max} = R_t(0) \geq R_t(\lambda)$ for all $\lambda \in [0, 1]$, it will be optimal to borrow enough to produce at the maximal capacity possible. For $\lambda > w$, this maximum is $\frac{w}{\lambda}$. Her profits would thus equal

$$\frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \quad (57)$$

where recall $x_t(\lambda) = (1 - \lambda)(1 + R_t(\lambda))$ is the amount a borrower is required to repay per each unit of resource she borrows. Since $R_t(\lambda) = \bar{R}_t$ for all $\lambda \in (\Lambda_t, 1)$, there would be no benefit to going to market $\lambda > \Lambda_t$: She would have to produce less at the same interest rate as in market Λ_t . The only case that remains is the interval of markets $\lambda \in [w, \Lambda_t]$. In that case, we can differentiate profits in (57) to get

$$\begin{aligned} \frac{d}{d\lambda} \left(\frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \right) &= -\frac{w}{\lambda^2} \left[(1 + y^* - x_t(\lambda)) + \lambda \frac{dx_t(\lambda)}{d\lambda} \right] \\ &= -\frac{w}{\lambda^2} \left[(1 + y^* - x(\lambda)) - \frac{\lambda(1 + \bar{R}_t)}{\int_x^{1+r_t^{\max}} dG_t(z)} \right] \\ &= -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)} \left[\int_x^{1+r_t^{\max}} (1 + y^* - x_t(\lambda)) dG_t(z) - \lambda(1 + \bar{R}_t) \right] \end{aligned}$$

Since $y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$, we have

$$\frac{d}{d\lambda} \left(\frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \right) < -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)} \left[\int_x^{1+r_t^{\max}} (z - x_t(\lambda)) dG_t(z) - \lambda(1 + \bar{R}_t) \right]$$

But for $\lambda \leq \Lambda_t$, the expression in brackets is equal to 0. Hence, borrowing in a market with $\lambda > w$ will be strictly dominated by borrowing in the market with $\lambda = w$. At the optimum, each entrepreneur borrow $1 - w$ at a rate of $R_t(w)$. ■

Proposition C3: *In an equilibrium where all markets are active, the equilibrium price of the asset will be given by $p_t = (1 - \varphi)e$*

Proof of Proposition C3: Condition (49) implies that all the resources of the young in cohort t will be used to either produce or to buy assets. From Proposition C2, we know that all entrepreneurs will produce at capacity, so the total amount used to produce is given by

$$\int_0^1 (2\varphi e) dw = 2\varphi e$$

This implies

$$p_t + 2\varphi e = (1 + \varphi)e$$

and so $p_t = (1 - \varphi)e$ as claimed. ■

Propositions C1-C3 do not require any restrictions on the distribution of d_t . When $d_t = d$, the return on the asset $1 + r_t$ will have a degenerate distribution with full mass at $\frac{d}{(1-\varphi)e}$. Substituting this into (52) reveals that $\tilde{x}(\lambda) = (1 - \lambda) \left(1 + \frac{d}{(1-\varphi)e} \right)$ for all λ , that $\frac{d\tilde{R}(\lambda)}{d\lambda} = 0$ for all λ , and the cutoff $\Lambda_t = 0$. Hence, when all markets are active, $R_t(\lambda) = \bar{R}_t = \frac{d}{(1-\varphi)e}$ for all $\lambda \in [0, 1)$ as described in the text. One equilibrium in which all markets are active if it entrepreneurs with wealth w borrow in market $\lambda = w$. But other equilibria in which all markets are active also exist.

When dividends follow a regime-switching process, then if $d_t = D$ at date t , z_t would be distributed as

$$z_t = \begin{cases} \frac{D}{(1-\varphi)e} & \text{w/prob } 1 - \pi \\ \frac{d}{(1-\varphi)e} & \text{w/prob } \pi \end{cases}$$

We can verify that this distribution implies that $\frac{dR_t(\lambda)}{d\lambda} < 0$ when $\lambda \leq \Lambda_t^D$. In particular, observe that (56) in Lemma C2 relies on the fact that $\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} z_t dG(z) \leq \int_0^{1+r_t^{\max}} z_t dG(z)$. But for the above distribution, the first expression is equal to $(1-\pi) \left(1 + \frac{D}{(1-\varphi)e}\right)$, which is strictly less than $1 + \frac{(1-\pi)D+\pi d}{(1-\varphi)e}$ which corresponds to the second expression. Hence, we can replace (56) with a strictly inequality, implying $\frac{dR_t(\lambda)}{d\lambda}$ is strictly negative for $\lambda \leq \Lambda_t^D$. This is in line with what we discuss in the text and depict in Figure 3.

Since Λ_t^D is the minimum value of λ at which $\frac{\tilde{x}_t(\lambda)}{1-\lambda} = 1 + \bar{R}_t^D$, we have

$$\frac{1}{1-\Lambda_t} \left[1 + \frac{D}{(1-\varphi)e} - \left(1 + \bar{R}_t^D\right) \frac{\Lambda_t}{1-\pi} \right] = 1 + \bar{R}_t^D$$

which, upon rearranging, yields

$$\Lambda_t^D = \frac{1-\pi}{\pi(1+\bar{R}_t^D)} \left(\frac{D}{(1-\varphi)e} - \bar{R}_t^D \right)$$

Since $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0, \Lambda_t^D)$, Proposition C2 implies only borrowers with wealth w borrow in market $\lambda = w$ for $w \in [0, \Lambda_t^D)$. Hence, $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$. By contrast, $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. However, we know that $f_t^p(\Lambda_t) > 0$, since borrowers with wealth $w = \Lambda_t^D$ will have to borrow in this market to borrow $1-w$. As for the amount borrowed to buy assets, $f_t^a(\lambda)$, we can deduce $f_t^a(\lambda)$ for $\lambda \in [0, \Lambda_t^D]$ from $R_t^D(\lambda)$, \bar{R}_t^D , and $f_t^p(\lambda)$ using (50). For $\lambda > \Lambda_t^D$, the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ at $\lambda = \Lambda_t^D$, combined with the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ for $\lambda > \Lambda_t^D$ from Lemma C2, implies that no agent would want to borrow to buy assets. So $f_t^a(\lambda) = 0$ for all $\lambda \geq \Lambda_t^D$. We can solve for \bar{R}_t^D as in the text.

C.3 Comparative Statics

Next, we consider equilibria where all markets above some floor $\underline{\lambda}$ are active. These results correspond to Propositions 8 and 9 in the text. The first result concerns how the equilibrium changes with $\underline{\lambda}$.

Proof of Proposition 8: In the text, we show that p^D and \bar{r}^D are increasing and decreasing in $\underline{\lambda}$, respectively. Here, we show that \bar{R}^D is decreasing in $\underline{\lambda}$. For any $\underline{\lambda}$, either \bar{R}^D equals \bar{r}^D or exceeds \bar{r}^D . Since the expected return on loans \bar{R}^D is continuous in $\underline{\lambda}$, it will suffice to show that \bar{R}^D is decreasing in $\underline{\lambda}$ when $\bar{R}^D > \bar{r}^D$.

When $\bar{R}^D > \bar{r}^D$, we have $\gamma^D = 1$, and the equilibrium conditions for R^D and Λ^D are given by

$$(1 - \Lambda^D) \bar{R}^D = \left[\frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \Lambda^D \left(\frac{1+\bar{R}^D}{1-\pi} - 1 \right) \right] \quad (58)$$

$$1 + \bar{R}^D = (1 - \varphi(1 - \underline{\lambda})) [1 + \bar{r}^D - \pi\Phi] + 2\varphi \int_0^1 \left[\min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max \{w, \underline{\lambda}\})] dw \quad (59)$$

If $\bar{R}^D > \bar{r}^D$, the floor $\underline{\lambda}$ must be below the cutoff Λ^D . For suppose $\underline{\lambda} \geq \Lambda^D$. Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected

return on the asset must be equal to ensure both the credit market and asset market clear. Since $\underline{\lambda} < \Lambda^D$, we can expand the integral term in (59) to obtain

$$\int_0^1 \left[\min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max\{w, \underline{\lambda}\})] = (1 + R(\underline{\lambda})) \left(\frac{1}{\underline{\lambda}} - 1 \right) \int_0^{\underline{\lambda}} w dw + \int_{\underline{\lambda}}^{\Lambda^D} (1 + R(w))(1 - w) dw + (1 + \bar{R}^D) \int_{\Lambda^D}^1 (1 - w) dw$$

We use the fact that $1 + R^D(\lambda) = \frac{1}{1-\lambda} \left[1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \frac{\lambda(1+\bar{R}_t^D)}{1-\pi} \right]$ to express the three integrals above as

$$(1 + R(\underline{\lambda})) \left(\frac{1}{\underline{\lambda}} - 1 \right) \int_0^{\underline{\lambda}} w dw = \left[1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \underline{\lambda} \left(\frac{1+\bar{R}^D}{1-\pi} - 1 \right) \right] \frac{\underline{\lambda}}{2} \quad (60)$$

$$\int_{\underline{\lambda}}^{\Lambda^D} (1 + R(w))(1 - w) dw = \int_{\underline{\lambda}}^{\Lambda^D} \left[1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \frac{w(1+\bar{R}_t^D)}{1-\pi} \right] dw \quad (61)$$

$$(1 + \bar{R}^D) \int_{\Lambda^D}^1 (1 - w) dw = \frac{1}{2} (1 + \bar{R}^D) (1 - \Lambda^D)^2 \quad (62)$$

We can write (58) and (59) more compactly as

$$\begin{aligned} h_1(\bar{R}^D, \Lambda^D) &= 0 \\ h_2(\bar{R}^D, \Lambda^D) &= 0 \end{aligned}$$

Totally differentiating this system of equations gives us the comparative statics of the equilibrium \bar{R}^D and Λ^D with respect to any variable a as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \bar{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \bar{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix} \begin{bmatrix} d\bar{R}^D/da \\ d\Lambda^D/da \end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a} \end{bmatrix}$$

Differentiating (58) and (59) using expressions (60)-(62) yields

$$\begin{aligned} \frac{\partial h_1}{\partial \bar{R}^D} &= 1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} & \frac{\partial h_1}{\partial \Lambda^D} &= \frac{\pi(1+\bar{R}^D)}{1-\pi} \\ \frac{\partial h_2}{\partial \bar{R}^D} &= 1 + \varphi \left[\frac{1}{1-\pi} (\Lambda^D)^2 - (1 - \Lambda^D)^2 \right] & \frac{\partial h_2}{\partial \Lambda^D} &= 0 \end{aligned}$$

When we evaluate comparative statics with respect to $\underline{\lambda}$, we now have

$$\begin{aligned} \begin{bmatrix} d\bar{R}^D/d\underline{\lambda} \\ d\Lambda^D/d\underline{\lambda} \end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \bar{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \bar{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}} \end{bmatrix} \\ &= \frac{\varphi}{\kappa} \begin{bmatrix} 0 & \frac{\pi}{1-\pi} (1 + \bar{R}^D) \\ 1 + \varphi \frac{(\Lambda^D)^2}{1-\pi} - \varphi (1 - \Lambda^D)^2 & - \left(1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} \right) \end{bmatrix} \begin{bmatrix} -\frac{D}{(1-(1-\lambda)\varphi)^2 e} \\ -\frac{2D(1+\Lambda^D\varphi)}{(1-(1-\lambda)\varphi)^2 e} - (1 + \pi\Phi) \end{bmatrix} \end{aligned}$$

where $\kappa = \frac{\pi(1+\bar{R}^D)}{1-\pi} \left(1 + \varphi \frac{(\Lambda^D)^2}{1-\pi} - \varphi (1 - \Lambda^D)^2 \right) > 0$. It follows that

$$\frac{d\bar{R}^D}{d\underline{\lambda}} = -\varphi \left(1 + \varphi \left[\frac{1}{1-\pi} (\Lambda^D)^2 - (1 - \Lambda^D)^2 \right] \right)^{-1} \left[\frac{2D(1+\Lambda^D\varphi)}{(1-(1-\lambda)\varphi)^2 e} + (1 + \pi\Phi) \right] < 0$$

Since \bar{R}^D is decreasing in $\underline{\lambda}$ whether $\bar{R}^D > \bar{r}^D$ or $\bar{R}^D = \bar{r}^D$, the claim follows. ■

Proposition 9 concerns how changing $\underline{\lambda}$ affects the expected costs of default $\gamma^D \Phi p^D$. Since we already know p^D is increasing in $\underline{\lambda}$, any changes in expected default costs occur entirely through γ^D . Our next result argues that there exists cutoffs Λ_0 and Λ_1 such that $d\gamma^D/d\underline{\lambda} = 0$ when $\underline{\lambda} < \Lambda_0$ or $\underline{\lambda} > \Lambda_1$. When $\Lambda_0 < \underline{\lambda} < \Lambda_1$, we only claim it must be decreasing for some $\underline{\lambda}$ in this interval.

Proof of Proposition 9: Define

$$\rho(\underline{\lambda}) = \frac{\bar{R}^D}{(1 - (1 - \underline{\lambda}) \varphi)}$$

Using the fact that $\frac{d\bar{R}^D}{d\underline{\lambda}} < 0$, we have

$$\frac{d\rho(\underline{\lambda})}{d\underline{\lambda}} = \frac{d\bar{R}^D/d\underline{\lambda} - \varphi\rho(\underline{\lambda})}{1 - (1 - \underline{\lambda}) \varphi} < 0$$

Since

$$\bar{R}^D/\bar{r}^D = [(1 - \pi) D + \pi d] \rho(\underline{\lambda})$$

it follows that the ratio \bar{R}^D/\bar{r}^D is decreasing in $\underline{\lambda}$. Hence, there exists a value $\Lambda_0 \geq 0$ such that $\bar{R}^D > \bar{r}^D$ for $\underline{\lambda} < \Lambda_0$ and $\bar{R}^D = \bar{r}^D$ for $\underline{\lambda} \geq \Lambda_0$. Since $\bar{R}^D > \bar{r}^D$ when $\underline{\lambda} < \Lambda_0$, then $\gamma^D = 1$ for $\underline{\lambda} < \Lambda_0$. It follows that expected default costs $\pi\gamma^D\Phi p^D = \pi\Phi p^D$ are increasing in $\underline{\lambda}$ in this region. A higher $\underline{\lambda}$ for $\lambda < \Lambda_0$ reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where $\underline{\lambda} \geq \Lambda_0$. Here, we know $\bar{R}^D = \bar{r}^D$. Substituting this into (58) yields

$$(1 - \Lambda^D) (1 + \bar{r}^D) = \left[1 + \frac{D}{(1 - (1 - \underline{\lambda}) \varphi) e} - \frac{\Lambda^D}{1 - \pi} (1 + \bar{r}^D) \right]$$

which, upon rearranging,

$$\Lambda^D = \frac{(1 - \pi)(D - d)}{(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d}$$

From this, we can conclude that $\Lambda^D \geq \underline{\lambda}$ if

$$\frac{(1 - \pi)(D - d)}{(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1 - \pi)(D - d) \geq \underline{\lambda} [(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d] \quad (63)$$

The RHS of (63) is a quadratic in $\underline{\lambda}$ with a positive coefficient on the quadratic term. The inequality is satisfied when $\underline{\lambda} = 0$ and violated when $\underline{\lambda} = 1$. This implies there exists a cutoff $\Lambda_1 \in (0, 1)$ such that $\Lambda^D > \underline{\lambda}$ if $\underline{\lambda} \in [0, \Lambda_1)$ and $\Lambda^D < \underline{\lambda}$ if $\underline{\lambda} \in (\Lambda_1, 1)$. We can deduce that $\Lambda_1 \geq \Lambda_0$ since by definition Λ_0 is the cutoff such that $\bar{R}^D = \bar{r}^D$ when $\underline{\lambda} \geq \Lambda_0$, yet at $\underline{\lambda} = \Lambda_1$ we have

$$\bar{R}^D = R(\Lambda^D) = R(\underline{\lambda}) = R(\Lambda_1)$$

By construction, we know that $R(\underline{\lambda})$ when $\underline{\lambda} = \Lambda_1$ is equal to \bar{r}^D . This implies $\Lambda_1 \geq \Lambda_0$.

When $\underline{\lambda} > \Lambda_1$ no agent will borrow to buy the asset, so $\gamma^D = 0$. Expected default costs are 0, and so the only effect of increasing $\underline{\lambda}$ is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where $\Lambda_0 < \underline{\lambda} < \Lambda_1$. We do not analyze this case in general. However, when $\Lambda^D = \underline{\lambda}$, the interest rate in all active markets would equal \bar{R}^D , since the only active markets are those with $\lambda \geq \underline{\lambda} = \Lambda^D$. Since $\underline{\lambda} \geq \Lambda_0$, we know that $\bar{R}^D = \bar{r}^D$ and so the interest rate in all active markets is \bar{r}^D . The equilibrium condition that determines γ^D is given by

$$\begin{aligned}
(1 + \bar{r}^D) &= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi \int_0^1 \left[\min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max\{w, \underline{\lambda}\})] dw \\
&= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi (1 + \bar{r}^D) \int_0^1 \left[\min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] dw \\
&= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi (1 + \bar{r}^D) [\underline{\lambda}/2 + (1 - \underline{\lambda}) - 1/2] \\
&= 1 + \bar{r}^D - \gamma^D (1 - (1 - \underline{\lambda})\varphi) \pi \Phi
\end{aligned}$$

Hence, when $\underline{\lambda} = \Lambda_1$, we have $\gamma^D = 0$. For $\underline{\lambda} < \Lambda_1$, however, $\gamma^D > 0$, since

$$\int_0^1 [1 + R^D(\max\{w, \underline{\lambda}\})] \left[\min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] dw$$

will be strictly greater than $\frac{1}{2} (1 + \bar{r}^D) (1 - \underline{\lambda})$. Hence, in the limit as $\underline{\lambda} \uparrow \Lambda_1$, we have $d\gamma^D/d\underline{\lambda} < 0$ expected default costs $\pi\gamma^D\Phi p^D$ must be decreasing in $\underline{\lambda}$ since this expression goes from a positive value to 0.

Finally, to show that this can generate a Pareto improvement, observe that increasing $\underline{\lambda}$ while dividends are high will make the initial old at date 0 better off given p_0^D increases. Cohorts born after dividends have fallen will be unaffected if $\underline{\lambda}$ is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs $E[d_{t+1}] - \Phi\pi\gamma^D p_t^D$ as well as the output produced by entrepreneurs. If Φ is sufficiently large and φ is small, we can promise these agents a higher expected consumption. ■

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