Recurrent Bubbles and Economic Growth*

Pablo A. Guerron-Quintana[†] Tomohiro Hirano[‡] Ryo Jinnai[§]

May 30, 2019

Abstract

We develop a novel model of recurrent bubbles with endogenous growth, infinitely-lived agents, and financial frictions. Expectations about future bubbles crowd out investment, thereby reducing economic growth, while bubbles, once materialized, promote economic growth. Their overall impact on economic growth and welfare depends on financial development and the frequency of bubbles. Moreover, we examine U.S. economic growth performance through the lens of our model, finding evidence of recurrent bubbles. The emergence and the bursting of the asset bubble in the 2000s are important to account for the acceleration and the slowdown in economic growth before and after the Great Recession.

1 Introduction

Ten years after the worst crisis since the Great Depression, economic observers seem to agree on a few points. First, an asset price bubble emerged in the years leading up to the crisis. Second, the implosion of this bubble triggered a financial crisis, resulting in a deep contraction in the economy, aka the Great Recession (Brunnermeier and Oehmke (2013)). Third, the post-financial-crisis recovery has been lackluster, with GDP growing about 1 percentage point slower than before the crisis (3% pre-crisis versus 2% post-crisis). Interestingly, Cerra and Saxena (2008), Blanchard,

*We are thankful to Dongho Song for extensive discussions and feedback. We also benefited from useful comments by Levent Altinoglu, Kosuke Aoki, Gadi Barlevy, Susanto Basu, Fernando Broner, Bernard Dumas, Andrew Foerster, Masashige Hamano, Takashi Kamihigashi, Michihiro Kandori, Nobuhiro Kiyotaki, Nan Li, Alberto Martin, Kiminori Matsuyama, Masaya Sakuragawa, Jose Scheinkman, Joseph Stiglitz, Jean Tirole, Vincenzo Quadrini, Rosen Valchev, Jaume Ventura, and seminar participants at Bank of Canada, Bank of Japan, Beijing University, Boston College, Canon Institute for Global Studies, CREI, Espol, Hitotsubashi University, Japan Center for Economic Research, Kobe University, Monash University, Norwegian Business School, Okayama University, Osaka University, RIETI, Shanghai Jiao Tong University, Tokai University, University of Birmingham, University of Tokyo, Waseda University, Wuhan University, and the NBER Summer Institute. A part of this work is supported by JSPS Kakenhi 18H00831.

[†]Boston College and Espol, pguerron@gmail.com

[‡]University of Tokyo, tomohih@gmail.com

[§]Hitotsubashi University, rjinnai@ier.hit-u.ac.jp

Cerutti, and Summers (2014), and Jorda, Schularick, and Taylor (2015) find that these features experienced by the U.S. in recent years are common to other financial crises around the world. Moreover, Jorda and coauthors, and Kindleberger (2001) show that these bubble-driven financial crises tend to be recurrent–i.e., recurring over time, with an interval of a few decades in many cases. In this paper, we take on the task of formulating a framework that rationalizes these empirical regularities: the existence of recurrent bubbles and their impact on economic growth, in particular, the growth slowdown after their collapse.

The framework is based on a tractable model of recurrent bubbles and endogenous growth. Investors are liquidity constrained à la Kiyotaki and Moore (Forthcoming), resulting in depressed investment and low growth. Bubbles may mitigate the problem of under-investment, which in our endogenous growth model enhances economic growth too. That is, by providing liquidity, bubbles crowd in investment. But if bubbles are helpful, their bursting brings economic distress. Just as in the data, the collapse of bubbles is followed by a sharp economic contraction and then prolonged low growth. The stagnation only ends when a new vintage of bubble emerges.

Following the literature, we formalize bubbles by using intrinsically useless "bubbly assets," which contribute neither to production nor to households' utility. The fundamental value of the assets is therefore zero. However, under some conditions, the price of the assets can be positive; that is, bubbles can exist in equilibrium. In addition, we formalize recurrent bubbles by introducing regime switching. There are two regimes: a "fundamental regime" and a "bubbly regime." No bubbles exist in the fundamental regime, and a new vintage of bubbly assets is provided to the households at the time of the regime change. The probability of regime switching is exogenous. Under these conditions, we analyze an interesting equilibrium in which bubbles exist in one regime but not in the other.

We find a novel crowding out effect of bubbles, which is unique to our formulation of recurrent bubbles. Households are long-lived in our model, and hence may experience multiple rounds of bubbles. Importantly, they fully anticipate the dynamics of these recurrent bubbles. That is, when bubbles exist, households anticipate their collapse, but when absent, they anticipate the emergence of bubbles. Crucially, these expectations about future bubbles affect households' decisions in both the fundamental and bubbly regimes. This is the critical difference between our work and the existing literature, including recent developments such as Kocherlakota (2009), Farhi and Tirole (2012), and Hirano and Yanagawa (2017), in which asset bubbles are not recurrent. A key insight is that expectations about future bubbles, especially about re-emergence, is a burden to economic growth. The mechanism is both simple and intuitive; it is essentially the wealth effect at work. Households will be wealthier when bubbles arise in the future. With this anticipation, households' consumption increases in both the fundamental and bubbly regimes, which crowds out investment and reduces economic growth in both regimes. This is a new crowding out effect of bubbles on investment as we discuss in the following section. Moreover, households are less eager to work in both regimes due to the wealth effect, further reducing economic growth. Because of these two effects, stagnation in economic growth occurs after the bubbles burst. That is, the economic growth rate after the bubbles collapse falls below the economic growth rate observed in the "fundamental equilibrium," in which the regime switching is turned off and the economy has been and will be in the fundamental regime forever. Such stagnation is not observed if bubbles are non-recurrent—i.e., if bubbles are not expected to re-emerge at all. As we discuss later, this result is also interesting in light of the existing literature.

Because of the crowding in effect of realized bubbles and the new crowding out effect of future bubbles, the overall impact of recurrent bubbles, especially high-frequency bubbles, depends on the level of financial development. If an economy's financial market is severely under-developed so that investors are seriously under-funded without bubbles, the crowding in effect tends to dominate. Hence, recurrent bubbles enhance both average growth and welfare over the long run compared to the same economy being in the fundamental equilibrium. In contrast, if the financial market is relatively developed, the crowding out effect may dominate, and recurrent bubbles reduce both average economic growth and welfare over the long run. If bubbles are more frequent, the crowding out effect becomes relatively stronger because households start to "count on" future bubbles more strongly. Therefore, high-frequency bubbles may not be desirable even in financially under-developed economies, not to mention financially developed ones.

In the quantitative application, dealing with our formulation of recurrent bubbles in DSGE models is intrinsically complicated. This is so because one must track the history of asset price bubbles, i.e., the dates of both the entire collapse and the re-emergence of bubbles, to characterize the current state of the economy. In our model, the states are capital, exogenous shocks, and a regime indicator. Since the economy switches between the two regimes, capital is regime dependent. But because of endogenous productivity, capital is a sufficient statistic for the history of bubbles. So once we detrend the model using capital, there is no regime dependence anymore and the equilibrium conditions depend only on the exogenous states of the economy. As a result, solving this model is tractable and the model is amenable to estimation.

We exploit these insights to map our model to the U.S. data for the period 1984 - 2017. In particular, we identify bubbles by exploiting the model's predictions that GDP growth and the credit-GDP ratio are high when bubbles exist but low in their absence. Then using data on those two observables, we find evidence of recurrent bubbles in the U.S. economy. Consistent with the model, estimated growth and credit are on average low during the fundamental (bubbleless) phase but high during the bubbly phase. Our estimation reveals that the U.S. economy spent the 1980s and 1990s in the fundamental regime. Since steady-state growth is weak in this regime, the bout of growth between 1995 and 2000 comes from a sequence of favorable exogenous productivity innovations. Moreover, contractionary productivity shocks were behind the 1991 and 2001 recessions. As credit increased rapidly in the 2000s, so did the probability of the economy being in the bubbly regime. By 2005, we estimate that there was a 50% chance that the economy was experiencing a bubbly episode. The bubble was in full swing in 2007. The Great Recession is initially the result of sharp adverse productivity and preference shocks followed by the collapse of the bubble, i.e., the return to the fundamental regime. We estimate that the economy has remained in the fundamental regime since then.

Counterfactual simulations allow us to evaluate the role of asset price bubbles in the U.S. economy. We do it in two ways. First, we consider a "no bubble by chance" scenario in which bubbles could emerge, which people knew, but did not materialize by chance. We find that, owing to the lack of the crowding in effect of realized bubbles, the economy would have grown at a lower rate in the 2000s in this scenario. Our estimation points to a 10 percentage point difference in 2011 between GDP in the actual economy (i.e., with a bubble) and GDP in the counterfactual simulation. Second, we consider a "no chance of bubbles" scenario, in which bubbles were impossible from the beginning and people knew of this impossibility; namely, it is the fundamental equilibrium. We find that, owing to the absence of the harmful crowding out effect of future bubbles, GDP growth would be higher in this scenario than in the actual economy, mainly because the U.S. has a developed financial system.

Our model provides a plausible explanation of the slowdown in growth over the past decades. As will become clear, bubbles, once realized, enhance growth in our framework. To the extent that the 2000s were a period associated with asset price bubbles, the collapse of these bubbles led inevitably to slower growth. Furthermore, growth will remain depressed until a new bubble emerges in the economy, a very unlikely event as of this writing according to our estimate. As a result, our parsimonious model can account for the post-Great Recession downward shift in the trend of the U.S. economic activity.

The rest of the paper proceeds as follows. Next, we highlight the contributions of our paper to existing literature. Then we describe the baseline model in Section 3. Section 4 discusses calibration. In Section 5, we discuss the impacts of recurrent bubbles on growth and welfare. The empirical results with a discussion of the last decades and the Great Recession are in Section 6. Section 7 concludes.

2 Related Work in the Literature

Our paper is in line with the literature on rational bubbles. The landmark papers are Samuelson (1958), Shell, Sidrauski, and Stiglitz (1969) (Section 3), Tirole (1985), Diba and Grossman (1988), and Farhi and Tirole (2012). In this literature, ours is closest to the studies on infinite horizon economies with imperfect financial markets. The seminal papers are Bewley (1980), Townsend (1980), Scheinkman and Weiss (1986), and Woodford (1990). These papers study deterministic fiat money (or government bonds) in an endowment economy when borrowing and lending are not allowed. Although they prove the existence of deterministic bubbles in infinite horizon economies, they do not necessarily show the necessary conditions explicitly for the reason asset bubbles can occur. Kocherlakota (1992) explicitly derived the necessary conditions for deterministic bubbles in an endowment economy when borrowing is allowed. Kocherlakota (2009) extends Kocherlakota (1992) to a production economy without growth, and examines the effects of land bubbles on production. Our model relies on the conditions that Kocherlakota (1992) derived; namely, liquidity constraints and a short-sale constraint are necessary for bubbles to exist.

Based on these seminal papers, we develop an endogenous growth model with financial frictions, and examine the impact of recurrent bubbles on long-run economic growth and welfare. In this regard, our paper is related to Hirano and Yanagawa (2017). There are, however, substantial differences. First, while we consider recurrent bubbles that are expected to arise and collapse recurrently in the future, Hirano and Yanagawa study the stochastic bubbles developed by Weil (1987). That is, a bubble is expected to collapse, but its reappearance is not expected at all. This difference is substantial because in our model, expectations about future bubbles affect households' decisions regarding consumption, labor supply, and investment through wealth effects, while in Hirano and Yanagawa (2017), those effects are completely absent. Second, the role of bubbles is also different. Hirano and Yanagawa emphasize the role of bubbles as speculative vehicles. Agents buy and sell bubbly assets mainly because they provide a high rate of return. In contrast, our paper emphasizes the role of bubbles as liquid assets; i.e., bubble assets can be sold quickly compared with illiquid capital. Our formulation of bubbles is based on Kiyotaki and Moore (Forthcoming) in which deterministic fiat money is modeled as a liquid asset.

As we mentioned in the introduction, our model with recurrent bubbles and infinitely lived agents has the new crowding out effect. A crowding out effect has been discussed in the existing literature focusing on the stochastic bubbles (Weil (1987), Kocherlakota (2009), Farhi and Tirole (2012), and Hirano and Yanagawa (2017)), but it is about the crowding out effect with respect to the present bubbles that already exist. In contrast, expectations about future bubbles are the source of the crowding out effect in our model, and interestingly, this effect is present in both the fundamental and bubbly regimes.

Moreover, because of the new crowding out effects together with the decrease in labor supply, growth stagnation occurs after the bubbles collapse. This result is in sharp contrast to Hirano and Yanagawa (2017). No growth stagnation occurs in their model, meaning that economic growth rate after the bubbles collapse is identical to the economic growth rate in the fundamental equilibrium. Recently, both Allen, Barlevy, and Gale (2017) and Biswas, Hanson, and Phan (2018) show that stagnation in the output level occurs after the bubbles burst in models without growth. But they show it by introducing amplification mechanisms worsening the severity of the recession. That is, Allen, Barlevy, and Gale introduce default costs exogenously associated with the collapse of bubbles. Biswas, Hanson, and Phan introduce downward nominal wage rigidities. In contrast, we show that even if the model abstracts from such mechanisms or other frictions— for example, price rigidities or fire-sale externalities— growth stagnation occurs by a standard and well-known channel in dynamic models: the intertemporal substitution of both goods and services.

Regarding recurrent bubbles, our paper is related to Gali (2014), Miao, Wang, and Xu (2015), and Kamihigashi (2011). In Gali (2014) and Miao, Wang, and Xu (2015), only a fraction of the existing bubbles collapses every period, and new bubbles are created right away so that the aggregate supply of bubbly assets is always positive. Moreover, these papers focus on a local analysis; i.e., there is no entire collapse of bubbles, and the model is linearized around the bubbly steady state. In our model, the entire collapse of bubbles occurs, and after that, the economy no longer stays in the neighborhood of the bubbly steady state but it is immediately pushed to the neighborhood of the bubbleless steady state. We capture these non-linear dynamics by perturbing the model around the regime-dependent steady states found in the original non-linear system. In this regard, our paper shares a similarity with Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Gertler and Kiyotaki (2015). In these papers, relatively large shocks to an economy cause the economy to jump far away from a steady state, producing highly non-linear effects. They emphasize that this non-linearity is important to account for financial crisis phenomena.

To the best of our knowledge, Kamihigashi (2011) would be the earliest paper on recurrent bubbles in infinite horizon economies. He provides sufficient conditions for a bubble process to be recurrent in a partial equilibrium model without production; neither investment nor endogenous labor supply decisions are considered. In contrast, we develop a dynamic general equilibrium model with production and endogenous growth, and examine the effects of recurrent bubbles on long-run economic growth and welfare.

The recurrent bubbles in Martin and Ventura (2012) are also related to ours, in the sense that there is an entire collapse of bubbles. However, our paper differs in important dimensions. First, their model is based on an overlapping generations model in which agents live for only two periods. They also assume that everyone supplies one unit of labor service inelastically in the young period, and consumes only in the old period. In this setting, anticipations about future bubbles do not matter for labor supply, consumption, and investment in the young period. As a result, their recurrent-bubble model is in essence simlilar to the stochastic-bubble model developed by Weil (1987). In contrast, our model has infinitely lived agents who anticipate both the emergence and the collapse of future bubbles, and this expectation about future bubbles affects labor supply, consumption, investment, and economic growth in both the fundamental and bubbly regimes. This point is also a crucial difference from Kamihigashi (2011) who studies bubbles in the endowment economy.

Our paper has an interesting new implication for the welfare impact of asset price bubbles. The classic argument on the topic is the consumption-smoothing effect (see Samuelson (1958), Bewley (1980), Scheinkman and Weiss (1986), and Hirano and Yanagawa (2017)). In Samuelson (1958), Bewley (1980), and Scheinkman and Weiss (1986), there is no saving technology, but when bubbly assets (fiat money in their models) appear, they provide a vehicle for savings, enhancing consumption smoothing. In Hirano and Yanagawa (2017), agents have an alternative to bubbly assets as a means of saving, but bubble assets provide an insurance against idiosyncratic shocks, thereby enhancing consumption smoothing. In both cases, asset bubbles are welfare improving even if bubbles are expected to collapse and reduce long-run economic growth. In our model, the welfare impact of asset price bubbles crucially depends on their effects on long-run economic growth because idiosyncratic shocks are perfectly shared among household members. Asset price bubbles can be either welfare improving or reducing depending on economic fundamentals. The frequency of bubbles also matters. The optimal frequency of bubbles exists, and is negatively correlated with the level of financial development. These results provide new insights into the normative aspect of asset price bubbles.

Our work is also related to Guerron-Quintana and Jinnai (Forthcoming) who examine causes of the post-war U.S. recessions through the lens of a DSGE model with both financial friction and endogenous growth. However, they do not introduce bubbly assets, and they linearize the model around the unique bubbleless steady state. As a result, they are unable to account for the growth slowdown after financial crises documented by Cerra and Saxena (2008), Blanchard, Cerutti, and Summers (2014), and Jorda, Schularick, and Taylor (2015). Theoretically, one can generate growth slowdown and acceleration by introducing regime-switching structural parameters to the endogenous growth model. But we find neither convincing empirical support nor theoretical justification for why the economic structure always changes in a particular way and why it coincides with the financial crisis.[1]

Furthermore, our work is related to studies on the role of financial development and growth as in Aghion, Howitt, and Mayer-Foulkes (2005). However, our focus is different from theirs. Namely, we focus on the provision of liquidity as a way to overcome under-developed financial systems, as well as the impact of recurrent bubbles on economic growth and welfare.

Our study of hysteresis is connected to previous work, such as Gali (2016), that studies hysteresis in labor markets and the design of monetary policy. We view our papers as complementary, since we highlight the role that bubbles may play in creating not only hysteresis but also super-hysteresis in economic activity. Finally, our work is related to the literature on the solution and estimation of Markov-switching models as in Farmer, Waggoner, and Zha (2009) and Hamilton (2016).

3 Model

Our description of the model consists of regimes, firms, households, and endogenous productivity.

[1] Guerron-Quintana and Jinnai (Forthcoming) document that there is no strong support for structural change in the financial market during or after the Great Recession; namely, many financial indicators temporarily deteriorated after the bankruptcy of Lehman Brothers, but have recovered in recent years.

3.1 Regimes

Let z_t denote a realization of the regime $z_t \in \{b, f\}$ where b and f denote the bubbly and fundamental regimes, respectively. Their defining features are the existence or the lack of bubbly assets, which are intrinsically useless, contributing to neither production nor households' utility directly. In the fundamental regime, there are no bubbly assets in the economy. When the regime switches to a bubbly one, M units of bubbly assets are created and given to households in a lump-sum way. There is no creation of bubbly assets in other contingencies. Bubbly assets last without depreciation as long as the economy stays in the bubbly regime. They disappear suddenly and completely once the regime switches back to the fundamental one. It is isomorphic to describe it as no depreciation but the sudden collapse of prices. We assume that z_t follows a Markov process satisfying

$$\Pr\left(z_t = f | z_{t-1} = f\right) = 1 - \sigma_f \tag{1}$$

and

$$\Pr\left(z_t = b | z_{t-1} = b\right) = 1 - \sigma_b. \tag{2}$$

3.2 Firms

Output is produced using capital and labor services denoted by KS_t^D and L_t^D, respectively. The production function is

$$Y_t = A_t \left(KS_t^D\right)^{\alpha} \left(L_t^D\right)^{1-\alpha}$$

where A_t is the technology level which agents in the economy take as given. Competitive firms choose KS_t^D and L_t^D to maximize profits defined as

$$Y_t - r_t KS_t^D - w_t L_t^D$$

where r_t is the rental price of capital and w_t is the wage rate. First-order conditions are standard.

3.3 Households

The economy is populated by a continuum of households, with measure one. Each household has a unit measure of members who are identical at the beginning of a period. During the period, members are separated from each other, and each member receives a shock that determines the role of the member in the period. A member will be an investor with probability $\pi \in [0, 1]$ and will be a saver with probability $1 - \pi$. These shocks are i.i.d. among the members and across time.

A period is divided into four stages: household's decisions, production, investment, and consumption. In the household's decision stage, all members of a household are together and pool their assets: n_t units of equities and $\tilde{m}_t$ units of bubbly assets. An equity is the ownership of a unit of capital. Aggregate shocks to exogenous state variables are realized. The capacity utilization rate u_t is decided. Because all the members of the household are identical in this stage, the household head evenly divides the assets among the members. The household head also gives contingency plans to each member as follows. If one becomes an investor, he or she spends i_t units of final goods to invest, and brings home the following items before the consumption stage: x_t^i units of final goods, n_{t+1}^i units of equity claims, and $\tilde{m}_{t+1}^i$ units of bubbly assets. Likewise, if the member becomes a saver, he or she supplies l_t units of labor, and brings home the following items before the consumption stage: x_t^s units of final goods, n_{t+1}^s units of equity claims, and $\tilde{m}_{t+1}^s$ units of bubbly assets. After receiving these instructions, members go to the market and remain separated from each other until the consumption stage.

At the beginning of the production stage, each member receives the shock determining his or her role in the period. Competitive firms produce final goods. Compensation for productive factors is paid to their owners. A fraction $\delta(u_t)$ of capital depreciates, which is increasing and convex in the utilization;

$$\delta(u_t) = \delta_0 + \frac{\delta_1}{1+\zeta} u_t^{1+\zeta}.$$

Note that the elasticity of $\delta(u_t)$ is constant at ζ; i.e., $\frac{u_t \delta''(u_t)}{\delta'(u_t)} = \zeta$ for all u_t.

Investors seek finance and undertake investment projects in the investment stage. Financing comes through different channels: own resources, selling of new and existing equity, and, if in the bubbly regime, selling of bubbly assets. Investors have access to a linear technology that transforms any amount of i_t units of final goods into i_t units of new capital. Asset markets close at the end of this stage. Members of the household meet again in the consumption stage. An investor consumes c_t^i units of final goods and a saver consumes c_t^s units of final goods.

These instructions must meet a set of constraints. First, they have to satisfy the intratemporal budget constraints; for an investor, it is

$$x_t^i + i_t + q_t \left(n_{t+1}^i - i_t - (1 - \delta(u_t)) n_t\right) + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \left(\tilde{m}_{t+1}^i - \tilde{m}_t\right) = u_t r_t n_t. \tag{3}$$

Here, q_t and $\tilde{p}_t$ denote prices of equities and bubbly assets, respectively. $\mathbf{1}$ is an indicator function to be discussed momentarily. In this paper, we say that asset price bubbles exist in period t if $\mathbf{1}_{\{z_t=b\}} \tilde{p}_t$ is strictly positive in period t. From equation (3), we see that the existence of asset price bubbles loosens up the investor's resource constraint contemporaneously, allowing her to invest more in the period. This is the source of the crowding in effect of asset price bubbles. The saver's constraint is

$$x_t^s + q_t \left(n_{t+1}^s - (1 - \delta(u_t)) n_t\right) + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \left(\tilde{m}_{t+1}^s - \tilde{m}_t\right) = u_t r_t n_t + w_t l_t. \tag{4}$$

Note that the saver is the counterpart in trading equity and bubbles. Loosely speaking, our setup represents situations in which investors are "borrowing" from savers. If bubbles exist, they increase the amount of this borrowing, which other things equal, opens the door to more investment and growth. Consistent with the empirical evidence in Bernanke (2018), the collapse of a bubble in our model leads to a disruption in the supply of this lending activity. We will exploit these predictions from the model to estimate the likelihood of bubbles in the data.

The indicator function in front of $\tilde{p}_t$ captures the idea that there is neither a spot nor a futures market for bubbly assets in the fundamental regime.[2] Without markets, no one can purchase bubbly assets, which is formally stated as follows:[3]

$$\mathbf{1}_{\{z_t=f\}}\tilde{m}^i_{t+1} = \mathbf{1}_{\{z_t=f\}}\tilde{m}^s_{t+1} = 0. \tag{5}$$

A feasibility constraint in the consumption stage is given by

$$\pi x^i_t + (1-\pi)\, x^s_t = \pi c^i_t + (1-\pi)\, c^s_t. \tag{6}$$

Following Kiyotaki and Moore (Forthcoming), we assume that an investor can issue new equity on, at most, a fraction ϕ of investment. In addition, she can sell, at most, a fraction ϕ of existing capital in the market too.[4] Effectively, these constraints introduce a lower bound to the capital holdings at the end of the period:

$$n^i_{t+1} \geq (1-\phi)\left(i_t + (1-\delta(u_t))\, n_t\right). \tag{7}$$

Following Shi (2015), we call it a liquidity constraint. A similar constraint applies to n^s_{t+1}, but we omit it because it does not bind in equilibrium; savers are net buyers of equities. We also omit non-negativity constraints for u_t, c^i_t, i_t, n^i_{t+1}, x^s_t, c^s_t, l_t, n^s_{t+1}, and $\tilde{m}^s_{t+1}$ for the same reason. Exceptions are both a short-sale constraint for investors

$$\tilde{m}^i_{t+1} \geq 0, \tag{8}$$

[2] We also assume that agents cannot make a contract contingent on future bubbles that can be attached to a new asset; i.e., future bubbles cannot be used as collateral.

[3] To justify this assumption, we could consider the following environment. Suppose that households need to pay transaction costs in order to investigate which assets bubbles are attached to in the future. If the transaction costs are sufficiently large, there will be no trading in the fundamental regime. Or suppose that there is a continuum of assets to which future bubbles can be attached. Households cannot know with certainty which assets bubbles can be attached to in the future. Under this setting, the probability that future bubbles can be attached to an asset is zero, and hence, the current price of that asset becomes zero. We thank Fernando Broner, Michihiro Kandori, and Albert Martin for their discussion on these interpretations.

[4] These two constraints are different in nature as Kiyotaki and Moore (Forthcoming) carefully distinguish. Our assumption that a single parameter ϕ governs both is just for simplicity.

and a borrowing constraint for investors

$$x_t^i \geq 0. \tag{9}$$

The household's problem is summarized as follows. A sequence of u_t, x_t^i, c_t^i, i_t, n_{t+1}^i, $\tilde{m}_{t+1}^i$, x_t^s, c_t^s, l_t, n_{t+1}^s, and $\tilde{m}_{t+1}^s$ is chosen to maximize

$$E_0\left[\sum_{t=0}^{\infty}\frac{\beta^t}{e^{b_t}}\left(\pi\frac{\left[c_t^i\right]^{1-\rho}-1}{1-\rho}+(1-\pi)\frac{\left[c_t^s\left(1-l_t\right)^{\eta}\right]^{1-\rho}-1}{1-\rho}\right)\right] \tag{10}$$

subject to (3), (4), (5), (6), (7), (8), (9), and the laws of motion for assets given by

$$n_{t+1} = \pi n_{t+1}^i + (1-\pi)\, n_{t+1}^s, \tag{11}$$

and

$$\tilde{m}_{t+1} = \pi\tilde{m}_{t+1}^i + (1-\pi)\,\tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}} M, \tag{12}$$

for all $t \geq 0$. The initial portfolio is $\{n_0, \tilde{m}_0\} = \left\{K_0, \mathbf{1}_{\{z_t=b\}} M\right\}$ where K_t denotes the capital stock in the economy in period t. b_t is a preference shock.

3.4 Learning-by-Doing

We assume that the technology level A_t is endogenous:

$$A_t = \bar{A}\left(K_t\right)^{1-\alpha} e^{a_t}.$$

a_t is an exogenous productivity shock and $\bar{A}$ is a scale parameter. Following Arrow (1962), Sheshinski (1967), and Romer (1986), we interpret the dependency of A_t on K_t as learning-by-doing; namely, knowledge is a by-product of investment, and in addition, it is a public good that anyone can access at zero cost. With it, the long-run tendency for capital to experience diminishing returns is eliminated.

3.5 Market Clearing

Competitive equilibrium is defined in a standard way; all economic agents optimize given prices, and the following market clearing conditions are satisfied:

$$n_{t+1} = K_{t+1}, \tag{13}$$

$$L_t^D = (1-\pi)\, l_t,$$

$$KS_t^D = u_t K_t,$$

and

$$\pi c_t^i + (1-\pi)\, c_t^s + \pi i_t = Y_t$$

for all t, and

$$\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = M$$

if $z_t = b$. Because the constraint (5) implies that $\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = 0$ holds if $z_t = f$, we have

$$\pi \tilde{m}_{t+1}^i + (1-\pi)\, \tilde{m}_{t+1}^s = \mathbf{1}_{\{z_t=b\}} M \tag{14}$$

for all t. The law of motion for capital is

$$K_{t+1} = (1-\delta(u_t))\, K_t + \pi i_t,$$

which automatically holds by Walras' law.

3.6 Solving the Household's Problem

It is convenient to solve the household's problem in two cases, depending on the tightness of the liquidity constraint.

3.6.1 When ϕ Is Loose

If the liquidity constraint is sufficiently loose, the equilibrium price of capital is equal to one, and the household is indifferent between investing in capital in-house and purchasing capital in the market. As a consequence, the liquidity constraint (7) does not bind. The borrowing constraint (9) does not bind either; if it did, the household could make it loose without affecting other constraints or the amount of equity holding at the end of the period by increasing x_t^i by $\Delta > 0$, decreasing x_t^s by $(\pi/(1-\pi))\,\Delta$, decreasing n_{t+1}^i by Δ, and increasing n_{t+1}^s by $(\pi/(1-\pi))\,\Delta$. We can also show that the equilibrium price of bubbly assets is zero in this case.

These observations allow us to summarize the constraints in a single equation:

$$\pi c_t^i + (1-\pi)\, c_t^s + n_{t+1} = [u_t r_t + (1-\delta(u_t))]\, n_t + w_t (1-\pi)\, l_t. \tag{15}$$

The first-order conditions in this case are

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) = 0,$$

and

$$1 = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\rho} \left(u_{t+1} r_{t+1} + 1 - \delta \left(u_{t+1} \right) \right) \right].$$

The first equation states that the marginal utility from consumption has to be equalized across members of the household. The second equation states that the marginal rate of substitution between leisure and consumption has to be equal to the wage. The third equation states that the marginal benefit of raising the capacity utilization rate has to be equal to its opportunity cost, which is the amount of depreciated capital at the margin. The fourth equation is the Euler equation.

3.6.2 When ϕ Is Tight

In the second case, the liquidity constraint is tight and it binds in equilibrium. The price of capital exceeds one in this case, because capital is used as "collateral" as well as the production factor. Moreover, $1 < q_t < 1/\phi$ is satisfied in equilibrium. These inequalities imply that investing in capital is profitable but investment cannot be made without down payments.

The inequality constraints (7) and (9) always bind for the following reasons. If (7) is not binding, households can increase their utility without violating any constraints or affecting their portfolio at the end of the period by increasing i_t by $\Delta > 0$, increasing n_{t+1}^i by $(q_t - 1)\Delta/q_t$, increasing both x_t^s and c_t^s by $(\pi/(1-\pi))(q_t - 1)\Delta$, and decreasing n_{t+1}^s by $(\pi/(1-\pi))((q_t-1)/q_t)\Delta$, which is a contradiction to the household's optimization. If (9) is not binding, households can relax (7) without violating any constraints or affecting their portfolio at the end of the period by decreasing x_t^i by Δ, increasing x_t^s by $(\pi/(1-\pi))\Delta$, increasing n_{t+1}^i by $(1/q_t)\Delta$, and decreasing n_{t+1}^s by $(\pi/(1-\pi))(1/q_t)\Delta$. This is a contradiction to the household's optimization because they can increase utility if (7) is not binding.

In addition, we can show that (8) holds with equality whenever asset price bubbles exist (i.e., $\mathbf{1}_{\{z_t=b\}}\tilde{p}_t > 0$). Suppose the opposite. Then, households can relax (7) without violating any constraints or affecting their portfolio at the end of the period by decreasing $\tilde{m}_{t+1}^i$ by Δ, increasing $\tilde{m}_{t+1}^s$ by $(\pi/(1-\pi))\Delta$, increasing n_{t+1}^i by $\tilde{p}_t\Delta/q_t$, and decreasing n_{t+1}^s by $(\pi/(1-\pi))(\tilde{p}_t/q_t)\Delta$. This is a contradiction to the household's optimization because they can increase utility if (7) is not binding.

Because (7), (8), and (9) hold with equality, the optimal investment is given by

$$i_t = \frac{\left[u_t r_t + \phi q_t \left(1 - \delta \left(u_t \right) \right) \right] n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t}{1 - \phi q_t}. \tag{16}$$

Substituting (7) and (16) into (11), we find

$$n_{t+1} = \pi \frac{1}{q_t} \left(1 + \lambda_t \right) \left[\left(u_t r_t + \phi q_t \left(1 - \delta \left(u_t \right) \right) \right) n_t + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t \tilde{m}_t \right] + \pi \left(1 - \phi \right) \left(1 - \delta \left(u_t \right) \right) n_t + \left(1 - \pi \right) n_{t+1}^s \tag{17}$$

where

$$\lambda_t \equiv \frac{q_t - 1}{1 - \phi q_t}. \tag{18}$$

Substituting (6) and (17) into (4), we find the budget constraint at the household level:

$$\begin{aligned} & \pi c_t^i + (1-\pi) c_t^s + q_t n_{t+1} + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t (1-\pi) \tilde{m}_{t+1}^s \\ = & \left[u_t r_t + (1-\delta(u_t)) q_t\right] n_t + \pi \lambda_t \left(u_t r_t + \phi q_t (1-\delta(u_t))\right) n_t \\ & + \mathbf{1}_{\{z_t=b\}} \tilde{p}_t (1+\pi\lambda_t) \tilde{m}_t + (1-\pi) w_t l_t. \end{aligned} \tag{19}$$

An important variable in this equation is λ_t, which Shi (2015) calls the liquidity service. It measures how much value an investor can create from a unit of liquidity. The reason is the following. An investor can create $1/(1-\phi q_t)$ units of capital from a unit of liquidity, which is the reciprocal of the marginal down payment. A fraction ϕ of the investment is equity financed, and the rest is added to the investor's portfolio, which is worth $(1-\phi) q_t/(1-\phi q_t)$ at the market price. Finally, substracting the costs of the investment from it, we find

$$\frac{(1-\phi) q_t}{1-\phi q_t} - 1 = \frac{q_t - 1}{1-\phi q_t} = \lambda_t.$$

Hence, λ_t is the marginal revenue from investment.

The household maximizes the expected utility (10) subject to the budget constraint (19), the accumulation rule for bubbly assets

$$\tilde{m}_{t+1} = (1-\pi) \tilde{m}_{t+1}^s + \mathbf{1}_{\{z_t=f, z_{t+1}=b\}} M,$$

and the absence of a bubbly-asset market in the fundamental regime

$$\mathbf{1}_{\{z_t=f\}} \tilde{m}_{t+1}^s = 0.$$

The first-order conditions are

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t \left(r_t - \phi q_t \delta'(u_t)\right) = 0, \tag{20}$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho} \left(u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} \left(u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1}))\right)\right) \right], \tag{21}$$

and

$$\mathbf{1}_{\{z_t=b\}} \tilde{p}_t = \mathbf{1}_{\{z_t=b\}} E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho} (1+\pi\lambda_{t+1}) \tilde{p}_{t+1} \mathbf{1}_{\{z_{t+1}=b\}} \right]. \tag{22}$$

The first two equations are the same as in the previous section, but the other equations are either different or new.

The third equation is the optimality condition for the capacity utilization rate, and the fourth equation is the Euler equation for capital. q_t appears in the second term in (20) because the opportunity cost of raising the capacity utilization rate is the value of depreciated capital at the margin. λ_t appears in the third term in (20) because the household head can provide additional liquidity to investors by raising the capacity utilization rate. λ_t appears in the right-hand side of (21) because capital is not only a production factor but also a means of providing liquidity to investors. Capital is valued based on both of these services.

Equation (22) is the Euler equation for the bubbly asset, and this is the key equation in our model. Two observations are worth noting. First, bubbles exist in period t only if there is a chance that the same bubbly assets will be traded at a strictly positive value in the next period. In other words, it is the resalability of bubbly assets in the future that justifies their positive prices in the present. Second, the parameter ϕ is absent in the equation. Bubbly assets are more liquid than capital, and with this advantage, savers may find the two assets indifferent at the margin even though bubbly assets are intrinsically useless.

Because $\mathbf{1}_{\{z_t=b\}}\tilde{m}_t = \mathbf{1}_{\{z_t=b\}}M$ holds in equilibrium,[5] (16) is rewritten as follows:

$$i_t = \frac{\left[u_t r_t + \phi q_t \left(1-\delta\left(u_t\right)\right)\right] K_t + \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M}{1-\phi q_t}. \tag{23}$$

The last term of the numerator ($\tilde{p}_t \mathbf{1}_{\{z_t=b\}} M$) is positive if and only if asset price bubbles exist ($\tilde{p}_t \mathbf{1}_{\{z_t=b\}} > 0$). This is the crowding in effect of realized bubbles; namely, they provide liquidity to investors, through which they increase gross investment. We discuss in the appendix that equation (23) plays a key role in determining whether bubbles are sustainable or not.

We can discuss the key insight of this study now, i.e., the crowding out effect of future bubbles.

[5] This is because the following relation holds:

$$\begin{aligned} \mathbf{1}_{\{z_t=b\}}\tilde{m}_t &= \mathbf{1}_{\{z_t=b\}}\left[\pi \tilde{m}_t^i + (1-\pi)\,\tilde{m}_t^s + \mathbf{1}_{\{z_{t-1}=f, z_t=b\}} M\right] \\ &= \mathbf{1}_{\{z_t=b\}}\left[\mathbf{1}_{\{z_{t-1}=b\}} M + \mathbf{1}_{\{z_{t-1}=f, z_t=b\}} M\right] \\ &= \mathbf{1}_{\{z_t=b\}} M. \end{aligned}$$

Substituting the budget constraints (19) forward, we derive an intertemporal budget constraint:

$$\begin{aligned}
& \pi c_0^i + (1-\pi)\, c_0^s + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{n,1} \cdots R_{n,t}} \left(\pi c_t^i + (1-\pi)\, c_t^s \right) \right] \\
= \; & \left(u_0 r_0 + [1-\delta(u_0)]\, q_0 + \pi \lambda_0 \left[u_0 r_0 + \phi q_0 (1-\delta(u_0)) \right] \right) n_0 \\
& + (1-\pi) \left(w_0 l_0 + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{n,1} \cdots R_{n,t}} w_t l_t \right] \right) \\
& + \pi \left(\lambda_0 \tilde{p}_0 \mathbf{1}_{\{z_0=b\}} M + E_0 \left[\sum_{t=1}^{\infty} \frac{1}{R_{1,t} \cdots R_{n,t}} \lambda_t \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M \right] \right),
\end{aligned} \tag{24}$$

where $R_{n,t}$ denotes the private return from capital, which is defined as

$$R_{n,t} \equiv \frac{u_t r_t + (1-\delta(u_t))\, q_t + \pi \lambda_t \left(u_t r_t + \phi q_t (1-\delta(u_t)) \right)}{q_{t-1}}.$$

The left-hand side of (24) is the present value of current and future consumption. The first term in the right-hand side is the value of current equity holdings. The second term is the present value of current and future labor income. Finally, the third term is the present value of current and future liquidity services provided by bubbly assets. If this term is positive, it relaxes the budget constraint, increasing consumption, decreasing the labor supply, and hence leaving fewer resources for investment. This is the crowding out effect of bubbles in our model.

Crucially important, the third term has both the current and future bubbles. Because the current bubbles appear in both (23) and (24), they have both the crowding in effect and the crowding out effect. Their overall impact on investment and growth is therefore uncertain but it is ultimately a quantitative question. In contrast, the future bubbles appear only in (24). They therefore have the crowding out effect alone. If bubbles are expected, they slow down current investment, and this effect exists in both the fundamental and bubbly regimes. We discuss this implication in detail in the following sections.

4 Calibration

As discussed above, our recurrent bubbles have both the crowding in and crowding out effects. To quantify their impact on growth and welfare, we turn to a quantitative analysis of the model. Table 1 summarizes the parameter values used in the rest of the paper. We set the discount factor at $\beta = 0.99$, the inverse of the intertemporal elasticity of substitution at $\rho = 1$, the capital share at $\alpha = 0.33$, and the elasticity of $\delta'(u_t)$ at $\zeta = 0.33$, following Comin and Gertler (2006). The probability of having an investment opportunity is set at $\pi = 0.06$, following Shi (2015).

The rest of the parameters are calibrated in the model. We assume that if there were no binding liquidity constraint, the growth rate of the economy would be 2% per annum, the hours worked

Parameter	Value	Calibration Target
β	0.99	Exogenously Chosen
ρ	1	Exogenously Chosen
ζ	0.33	Exogenously Chosen
α	0.33	Capital Share=0.33
π	0.06	Shi (2015)
δ_0	0.001	Frictionless Growth $g^4 = 1.02$
$\delta_1 u^{1+\zeta}$	0.065	Frictionless Depreciation $\delta(u) = 0.05$
η	2.67	Frictionless Hours $l = 0.27$
$\bar{A}u^{\alpha}$	0.49	Equilibrium Condition
u	1	Normalization

Table 1: Parameters and Calibration Targets

would be 27% of the available time, and the depreciation rate would be 5% per quarter along the balanced growth path. We then solve for the three parameters δ_0, $\delta_1 u^{1+\zeta}$, and η jointly. We find the value of $\bar{A}u^{\alpha}$ from the equilibrium condition. We set $u = 1$, which is just a normalization.

One may find that the targeted depreciation rate (5% per quarter) is high, but remember that this is the depreciation rate in an extreme situation in which ϕ is so large that the liquidity constraint never binds. Previous studies in the literature assume a smaller ϕ (Kiyotaki and Moore (Forthcoming) and Shi (2015)). If we follow Kiyotaki and Moore and set it at $\phi = 0.19$ in our calibrated model, the implied depreciation rate is 2.4% per quarter. However, we are agnostic about the value of ϕ at this point. We show the comparative statics with respect to this parameter in the following section.

5 Comparative Statics

In this section, we discuss the impact of bubbles on growth and then on welfare.

5.1 Growth in Fundamental Equilibrium

Figure 1 shows how the degree of the liquidity constraint, ϕ, influences the speed of economic growth along the balanced growth path denoted by g. We assume that both productivity and preference shocks are constant at $a_t = b_t = 0$ for all t in this exercise. The blue line shows the result in the equilibrium in which bubbly assets are never traded at a positive price. Such an equilibrium always exists because bubbly assets are intrinsically useless. We call it the fundamental equilibrium, because it is essentially the same as the economy always being in the fundamental regime.

The blue line becomes flat on the right part of the figure, showing that the growth rate is constant once ϕ reaches a certain threshold. Beyond this point, the liquidity constraint does not bind because investors obtain the desired level of liquidity from equities alone. On the left

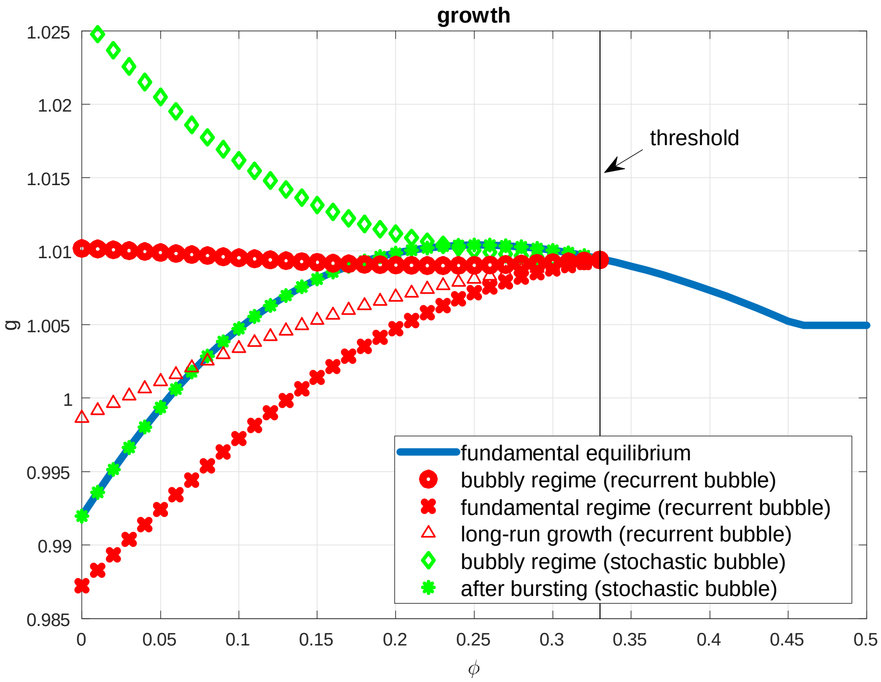

Figure 1: Liquidity and Growth in Recurrent-Bubble Model

part, the growth rate is influenced by the degree of the liquidity constraint. The relation is not only concave but also non-monotonic. We interpret ϕ as the degree of financial development in the economy, because this parameter governs how many resources investors can borrow using capital as collateral. The figure shows that neither too under-developed nor too advanced financial markets are beneficial for growth, but growth is maximized in an intermediate stage of financial development.

The source of concavity is explained as follows. As shown in Figure 2, the economy's investment relative to capital stock increases with ϕ. This is not surprising because a large ϕ relaxes the financial friction. But it is less obvious that investment-to-capital is not only increasing but also concave in ϕ. The price of capital q plays an important role here. As seen in the same figure, the price of capital is one if liquidity is plentiful. Remember that capital is nothing but a production factor there. However, if the liquidity constraint binds, the price of capital exceeds one. This is so because capital is now a production factor as well as the source of liquidity. Because the value of liquidity is high if it is limited, the price of capital is inversely related to ϕ.

This inverse relation between q and ϕ has an important consequence for investment. Because an investor can sell a fraction ϕ of capital at the market price q, the amount of liquidity an investor can obtain from a unit of capital held at the beginning of the investment stage is $\phi \times q$. The inverse relation between q and ϕ means that a marginal increase in ϕ delivers a sizable amount of liquidity to investors when ϕ is low (hence high q), and vice versa. This second-order effect makes investment and the growth rate of the economy concave in ϕ.

To understand the non-monotonicity, it is important to distinguish the net investment from the gross invesment. Remember that the capital depreciation rate is endogenous; it is a function of the capacity utilization rate u_t. The capacity utilization rate increases with ϕ as shown in Figure 2. This is because households care for the value of depreciated capital; if capital is cheap, they are less reluctant to raise the utilization rate because the opportunity cost is low. Because q is inversely related to ϕ, households choose a high utilization rate when ϕ is large, resulting in a high depreciation rate. It is slightly convex in ϕ because of the convexity of $\delta(\cdot)$. Taken together, the growth-enhancing effect of ϕ diminishes with ϕ (concave) and can even be negative (non-monotonic) because its impact on gross investment is concave and its impact on the depreciation rate is convex.

5.2 Growth with Stochastic Bubble

Now let us analyze the impact of bubbles on economic growth. We first look at the so-called stochastic bubble. Let us assume that bubbles exist at the beginning of the history, i.e., $\mathbf{1}_{\{z_0=b\}}\tilde{p}_0 > 0$. This initial bubble, however, bursts with a positive probability, triggered by the regime switch. After the bursting, there is no re-emergence of bubbles, even though the bubbly regime is revisited. This is one of the multiple equilibria in our model economy. Alternatively, we can think of it as a bubbly equilibrium in an economy in which the fundamental regime is an absorbing state ($\sigma_f = 0$). They are isomorphic. This kind of bubble, bursting stochastically after which the economy is permanently bubbleless, is studied in the pioneering work of Weil (1987).

We assume that the probability of the regime switch is 1.5% per quarter, meaning that the expected duration of the bubble is about 16.5 years. This duration is not unusual in the literature; for example, Hirano and Yanagawa (2017) analyze stochastic bubbles with an average duration varying from 12.5 to 100 years. Arguably long duration has been assumed in the literature because bubbles are not supported as an equilibrium outcome if they are too short-lived. The intuition is simple; no one buys bubbles if he or she knows that collapse is just around the corner. Instead, buyers must believe that bubbles are reasonably durable, and it is ultimately these buyers' beliefs that we model as the probability of the regime switch.

The green circles and crosses in Figure 1 show the implied growth rates in the stochastic-bubble equilibrium. There are two plots because growth rates are regime dependent. The bubbly equilibrium exists only if the liquidity constraint is sufficiently tight. The vertical line in the figure shows the threshold value for the existence of bubbles.

The economic growth in the initial bubbly regime is generally higher than the one after the bubble bursts. The key for this result is the intertemporal substitution or, more precisely, the inter-regime substitution. As shown in Figure 2, households work harder and invest more in the bubbly regime than in the fundamental one. The bubbly regime is a favorable time for investment,

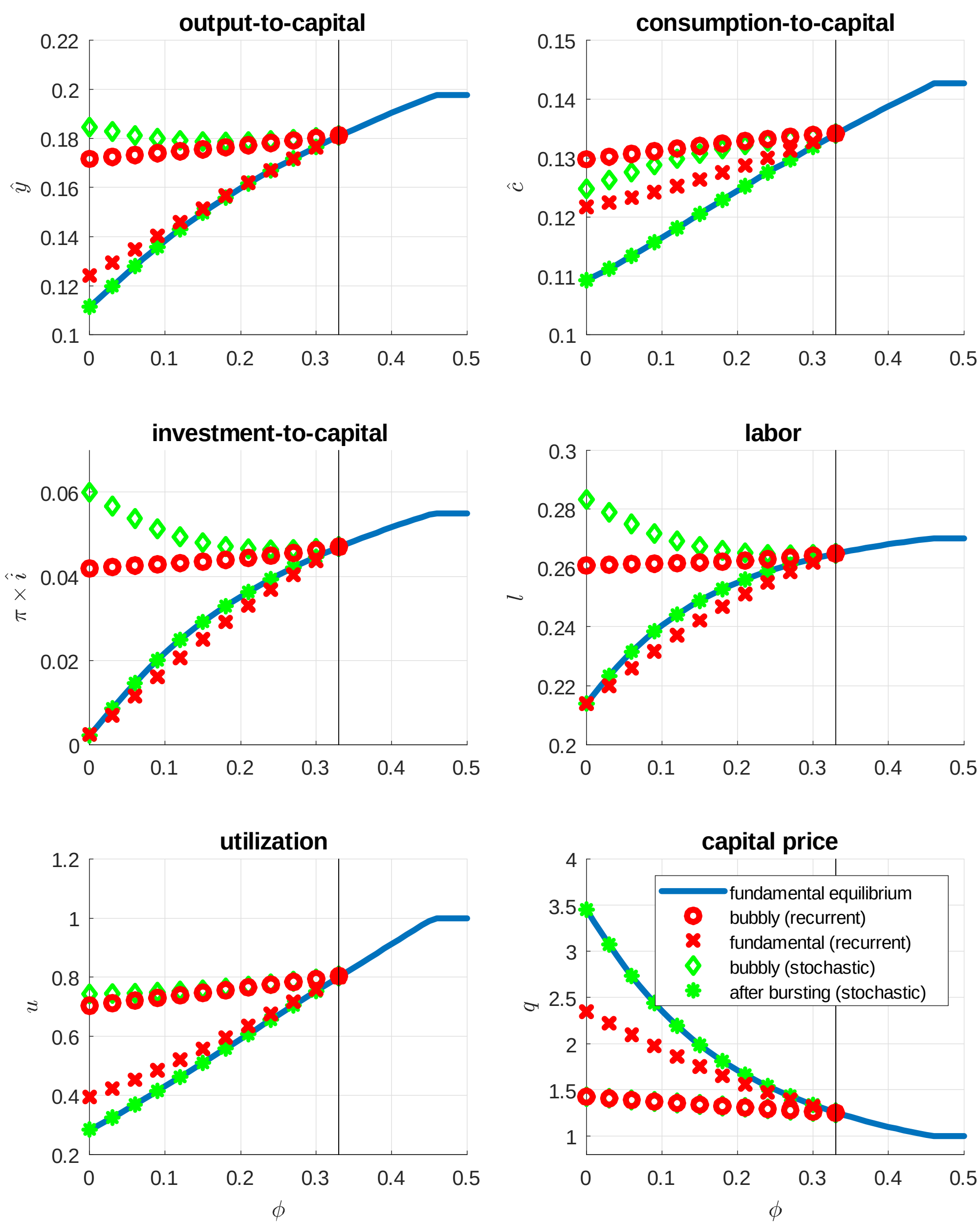

Figure 2: Effects of Liquidity in Recurrent-Bubble Model

and households, recognizing it, optimally allocate both time and resources not only across time but also across regimes. This is the crowding in effect of realized bubbles.[6]

There is no growth stagnation after the bubbles burst, or in other words, the growth rate after the collapse of bubbles is identical to the growth rate in the fundamental equilibrium. To undertand this result better, let us consider the following thought experiment. Suppose that the economy had been in the fundamental equilibrium for $t < 0$, but it moved unexpectedly and for reasons outside the model to the stochastic-bubble equilibrium in period 0. Then the bubble collapses in period $T > 0$. The question is whether the growth rate after the bubbles collapse ($t \geq T$) is lower than the growth rate before the bubbly episode begins ($t < 0$), not the growth rate in the interim bubbly episode ($0 \leq t \leq T - 1$). The answer is obviously no, but the growth rates before and after the bubbly episode are identical. The bubbly period is nothing but a temporary deviation. Interestingly, we see growth stagnation if bubbles are recurrent.

5.3 Growth with Recurrent Bubble

To analyze recurrent bubbles, we assume that the probabilities of regime switches are 1.5% per quarter in both directions, but the results are robust to other probabilities. Furthermore, we require the price of bubbly assets to always be positive whenever they exist ($\tilde{p}_t > 0$ if $z_t = b$). We call this equilibrium the recurrent-bubble equilibrium because bubbles occur repeatedly. The red circles and crosses in Figure 1 show regime-dependent growth rates in this situation. As in the stochastic-bubble equilibrium, the recurrent-bubble equilibrium exists only if the liquidity constraint is sufficiently tight.

The economic growth in the bubbly regime is higher than that in the fundamental regime in the recurrent-bubble equilibrium. This inter-regime growth differential is the result of the crowding in effect of realized bubbles we discussed in the previous section. When we compare the recurrent-bubble equilibrium and the stochastic-bubble equilibrium, we see that growth rates in the former equilibrium are generally lower than those in the latter equilibrium, conditional on being in the same regime. This inter-equilibrium growth differential is the result of the crowding out effect of future bubbles, which we will discuss momentarily.

Note that in the recurrent-bubble equilibrium, growth stagnation occurs after the bubbles burst; i.e., the economic growth in the fundamental regime (red cross) is consistently lower than that in the fundamental equilibrium (blue line). This result is interesting for at least two reasons. First, the environments are objectively identical; in both cases, no asset price bubbles exist at the time of the comparison. Second, we do not see this property for the stochastic-bubble equilibrium.

Growth stagnation occurs because the crowding out effect of future bubbles. As we see in

[6] As we discuss in the previous section, there is a crowding out effect of realized bubbles too; see equation (24) and the discussion there. The fact that the growth rate in the initial bubbly regime is generally higher than the growth rate after the collapse of the stochastic bubble means that the crowding in effect of realized bubbles generally dominates the crowding out effect of realized bubbles.

Figure 2, people consume more, work less (spend more time on leisure), and invest less in the fundamental regime of the recurrent-bubble equilibrium than in the fundamental equilibrium. They understand that future bubbles will make them richer, and this expectation makes people lazy, loosely speaking. The capacity utilization rate reduces net investment too. Namely, the price of capital is low if people expect the bubbles to re-emerge (the bottom-right panel of Figure 2) because bubbles provide liquidity to the economy, diluting the collateral value of capital. The low price of capital leads to a high capacity utilization rate (the bottom-left panel of Figure 2), slowing down the speed of capital accumulation as well as economic growth.

Growth stagnation has an important implication for long-run (unconditional) growth. To see this point, let us conduct a thought experiment analogous to the one we did in the previous section with stochastic bubbles. Suppose that the economy had been in the fundamental equilibrium in period $t < 0$, but it moved unexpectedly and for reasons outside the model to the recurrent-bubble equilibrium in period 0. The economy was in the bubbly regime in period 0, and the initial bubble collapses in period $T > 0$. We conduct this thought experiment in the economy with $\phi = 0.1$. We see in Figure 1 that the growth rate during the initial bubbly episode ($0 \leq t \leq T - 1$) is higher than the growth rate before the bubbly episode begins ($t < 0$). But the economy has the growth stagnation after the collapse, and it continues until the new bubbly episode begins. Hence, the overall impact of recurrent bubbles on the long-run growth is not obvious; we have to factor in the effect of each regime on growth. The red triangles in Figure 1 calculate such a thing, i.e., the long-run growth in the recurrent bubble equilibrium.[7] At $\phi = 0.1$, it is slower than the growth rate in the fundamental equilibrium. Hence, even though the growth rate is temporarily higher in the initial bubbly episode than before, the recurrent bubbles are harmful to growth in the long run. This result is a consequence of the growth stagnation; if we calculate the unconditional growth rate in the stochastic-bubble equilibrium in the same manner, it is identical to the growth rate in the fundamental equilibrium.

Figure 1 shows that economies with low ϕ grow faster in the long run if they are in the recurrent-bubble equilibrium than in the fundamental equilibrium. But the growth in the recurrent-bubble equilibrium is bumpy, disrupted by the occasional bursting of bubbles. Were the same economy in the fundamental equilibrium, the pace of economic growth would be slow but stable on average. This result is reminiscent of Ranciere, Tornell, and Westermann (2008), who document that countries that have experienced occasional financial crises have grown faster on average. Our model is consistent with their findings if we interpret the bursting of bubbles as a financial crisis at least for countries with small ϕ. However, for more advanced economies, in which investors obtain funds relatively easily, our model provides a different prediction. In such an economy, recurrent bubbles

[7] The speed of long-run growth $\bar{g}$ is given by

$$\bar{g} = g_{fr}^{\frac{\sigma_b}{\sigma_f + \sigma_b}} g_{br}^{\frac{\sigma_f}{\sigma_f + \sigma_b}}$$

where g_{br} and g_{fr} denote the speed of economic growth in the bubbly and fundamental regimes, respectively.

are harmful to economic growth in the long run.

5.4 Welfare Analysis

We discuss the welfare impact of the recurrent bubbles in this section. Our welfare measure is defined as follows. First, we rewrite the utility function (10) in the recursive form,

$$V_t = (1-\beta)\left\{\log\left[c_t\right] + (1-\pi)\,\eta\log\left[1-l_t\right]\right\} + \beta E_t\left[V_{t+1}\right].$$

Here, c_t is the common consumption level across members of the household ($c_t \equiv c_t^i = c_t^s$), which is an implication of the log utility. We keep assuming that $a_t = b_t = 0$ for all t in this section. Because both the continuation utility value V_t and the consumption level c_t have trends, we detrend them and rewrite the equation as follows:

$$\hat{V}_t = (1-\beta)\left\{\log\left[\hat{c}_t\right] + (1-\pi)\,\eta\log\left[1-l_t\right]\right\} + \beta\log\left[g_t\right] + \beta E_t\left[\hat{V}_{t+1}\right], \tag{25}$$

where $\hat{V}_t$ and $\hat{c}_t$ are defined as $\hat{V}_t \equiv V_t - \log K_t$ and $\hat{c}_t \equiv c_t/K_t$ respectively, and g_t is the capital growth $g_t \equiv K_{t+1}/K_t$. $\hat{V}_t$ is our welfare measure.[8]

The solid blue line in Figure 3 plots the level of $\hat{V}_t$ in the fundamental equilibrium as a function of ϕ, which is given by

$$\hat{V}_{fe}(\phi) = \log\left[\hat{c}_{fe}(\phi)\right] + (1-\pi)\,\eta\log\left[1-l_{fe}(\phi)\right] + \frac{\beta}{1-\beta}\log\left[g_{fe}(\phi)\right]. \tag{26}$$

The subscript fe denotes the fundamental equilibrium. Without loss of generality, we subtract $\hat{V}_{fe}(1)$ from $\hat{V}_{fe}(\phi)$ before plotting it so that it takes a value of zero in the case with sufficiently large ϕ.

The solid blue line in Figure 3 resembles the solid blue line in Figure 1, which suggests the importance of economic growth as a determinant of welfare. We confirm this observation using a factor decomposition. Namely, we vary the detrended level of consumption, hours worked, and economic growth one by one, while keeping the other two variables constant at their values in the environment in which liquidity is abundant. We plot the welfare in each of these exercises in red squares (consumption contribution), stars (leisure contribution), and diamonds (growth contribution), respectively.[9] If they are added up vertically, we obtain the solid blue line again. The consumption contribution monotonically increases with ϕ, but the leisure contribution decreases with it. This is because people not only consume more but also work longer as ϕ gets larger, as shown in Figure 2. With these two margins offsetting each other, the rate of economic growth emerges as the crucial factor for welfare.

[8] We borrow this welfare concept in a non-stationary setup from Schmitt-Grohe and Uribe (2005).

[9] For example, the red squares plot $\log\left[\hat{c}_{fe}(\phi)\right] - \log\left[\hat{c}_{fe}(1)\right]$ as a function of ϕ.

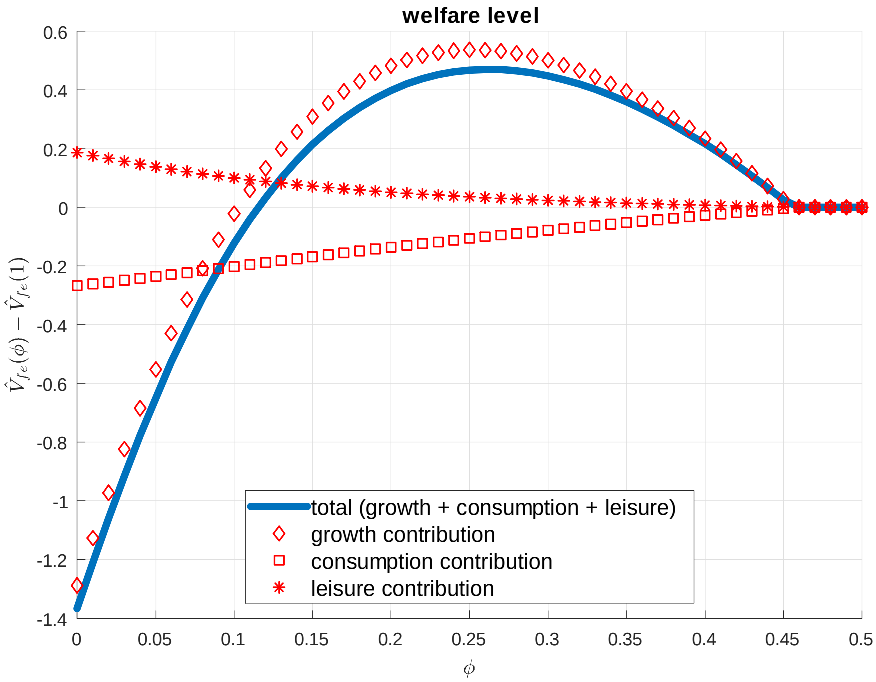

Figure 3: Liquidity and Welfare in Fundamental Equilibrium

Figure 4 plots the welfare levels in the recurrent-bubble equilibrium. We first calculate the regime-dependent welfare levels by solving the following equations;

$$\begin{pmatrix} \hat{V}_{fr} \\ \hat{V}_{br} \end{pmatrix} = \begin{pmatrix} (1-\beta)\left\{\log\left[\hat{c}_{fr}\right] + (1-\pi)\,\eta\log\left[1-l_{fr}\right]\right\} + \beta\log\left[g_{fr}\right] \\ (1-\beta)\left\{\log\left[\hat{c}_{br}\right] + (1-\pi)\,\eta\log\left[1-l_{br}\right]\right\} + \beta\log\left[g_{br}\right] \end{pmatrix} + \begin{pmatrix} 1-\sigma_f & \sigma_f \\ \sigma_b & 1-\sigma_b \end{pmatrix}\begin{pmatrix} \beta\hat{V}_{fr} \\ \beta\hat{V}_{br} \end{pmatrix}$$

where the subscripts fr and br denote the fundamental and bubbly regimes, respectively. Dependence on ϕ is omitted for brevity. We then calculate the unconditional welfare level in the recurrent-bubble equilibirum by

$$\hat{V}_{be} \equiv \frac{\sigma_b}{\sigma_b+\sigma_f}\hat{V}_{fr} + \frac{\sigma_f}{\sigma_b+\sigma_f}\hat{V}_{br}. \tag{27}$$

They are shown in the graph as red triangles ($\hat{V}_{be}$), crosses ($\hat{V}_{fr}$), and circles ($\hat{V}_{br}$). There are both a similarity to and a difference from their growth counterparts plotted in Figure 1. The similarity is the relative positions of the red triangles and the solid blue line. The red triangles are above the blue line in the leftmost part in both figures, meaning that for the economies with a weak financial system, recurrent bubbles are not only growth enhancing but also welfare improving in the long run. Bubbles increase volatilities of the economy, but the welfare loss from this channel is minor

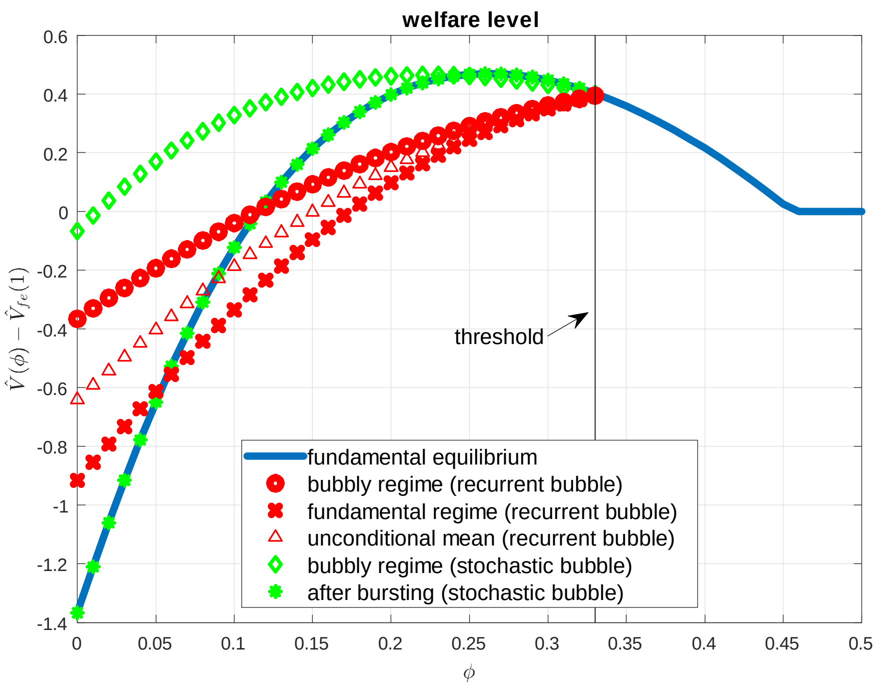

Figure 4: Liquidity and Welfare in Recurrent-Bubble Model

compared to the welfare gain from the boosted long-run growth. In contrast, recurrent bubbles reduce welfare in the long run if investors can relatively easily obtain funds without bubbles. This is not surprising because bubbles reduce the long-run growth and increase the volatility in such an economy.

The difference is in the distance between the outcomes in the bubbly regime (red circles) and in the fundamental regime (crosses). One can see that this distance is compressed in Figure 4 compared to Figure 1. Expectations are crucial for this result. The bubbly-regime welfare is relatively low, although growth in the same regime is relatively high because people anticipate that the bubble will eventually collapse. Similarly, people are not depressed in the fundamental regime despite the poor growth performance in the same regime because they expect the bubbles to re-emerge.

Figure 4 also shows the regime-dependent welfare levels in the stochastic-bubble equilibrium. The bubbly-regime welfare in this equilibrium is higher than the bubbly-regime welfare in the recurrent-bubble equilibrium. This result is interesting because the economy in the stochastic-bubble equilibrium has fewer bubbly episodes in the future (in fact, none) than the economy in the recurrent-bubble equilibrium. But remember that although realized bubbles provide extra liquidity, expectation about future bubbles crowds out investment in both the fundamental and the bubbly regimes. The bubbly-regime welfare in the stochastic-bubble equilibrium is especially high because it is in the special situation in which the crowding in effect of realized bubbles exists

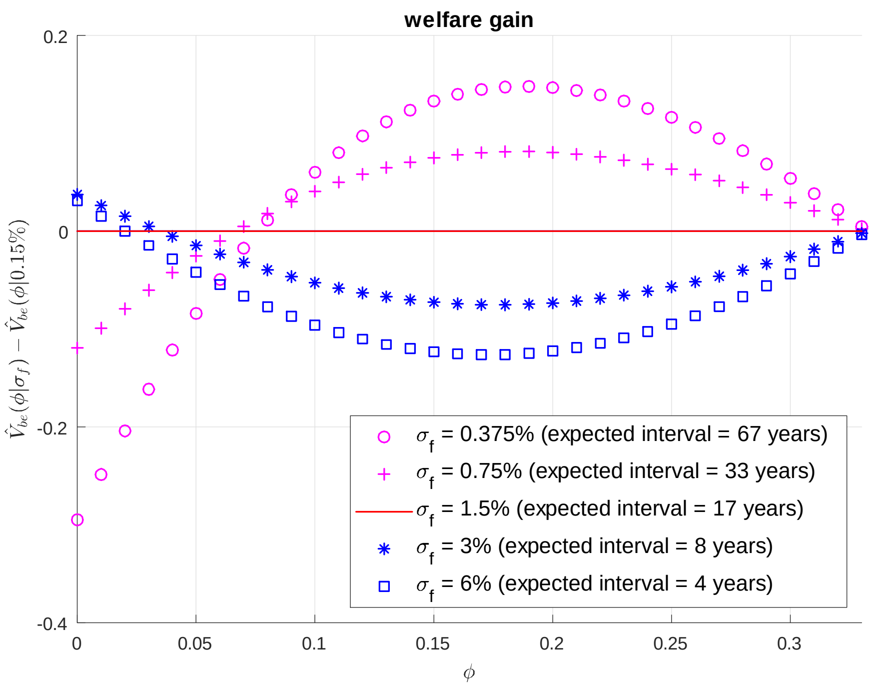

Figure 5: Frequency of Bubbles and Welfare

but the crowding out effect of future bubbles does not.

The tradeoff between the crowding in effect of realized bubbles and the crowding out effect of future bubbles becomes even more transparent by analyzing the welfare impact of high-frequency bubbles.[10] Specifically, we change σ_f while keeping the other parameters, including σ_b, constant. Hence, bubbles emerge at different frequencies across simulations in the otherwise identical economies. Importantly, the expected duration of each bubbly episode is constant. Results are shown in Figure 5, where we plot the welfare gain of having high- or low-frequency bubbles relative to the benchmark calibration $\sigma_f = 1.5\%$. Namely, we plot $\hat{V}_{be}(\phi|\sigma_f) - \hat{V}_{be}(\phi|1.5\%)$ where $\hat{V}_{be}(\phi|\sigma_f)$ is the unconditional welfare level in the recurrent-bubble equilibrium defined by (27). Blue stars and squares show the welfare gains of high-frequency bubbles ($\sigma_f > 1.5\%$), pink pluses and circles show the welfare gains of low-frequency bubbles ($\sigma_f < 1.5\%$), and the red line shows their counterpart of the benchmark calibration which is trivially zero.

We see a shape resembling the profile of a fish. It consists of the two intersecting arcs whose ends on the left side extend beyond the meeting point. But the upper and lower arcs are not perfectly symmetric with respect to the horizontal axis. Instead, the left crossing point is lower than the right. As a result, there is a parameter region in which not only the lower arc (blue) but also the upper arc (pink) sinks under the horizontal axis. For this parameter region, neither high-frequency nor low-frequency bubbles are preferred to the benchmark calibration. This result

[10] The authors thank Jean Tirole for the discussion guiding us to this exercise.

and the positions of the pink and blue arcs imply that the optimal (welfare-maximizing) frequency of bubbles decreases with the level of financial development. If the economy's financial system is severely under-developed, high-frequency bubbles are preferred because they can mitigate the liquidity shortage, the major growth bottleneck. But as the financial market gradually develops, lower-frequency bubbles start to be preferred, because the liquidity shortage becomes a less urgent issue, while the crowding out effect of future bubbles emerges as a new problem. This crowding out effect gets weaker as bubbles become less frequent because households count on bubbles less if they are unlikely to occur.

6 Taking the Model to the Data

As an empirical application, we use our model to revisit the economic performance of the U.S. over the last three decades. Specifically, we use quarterly U.S. data on the growth rate of GDP and the credit-to-GDP ratio for the period 1984.Q1 - 2017.Q4 to estimate the likelihood of bubbles as well as the paths of supply and preference shocks in our model (see the appendix for a detailed explanation of the data). We choose these observables because our model has sharp predictions about their behavior when bubbles are present in the economy.

Growth is high when bubbles exist as shown in Figure 1, but our model also predicts that the credit-to-GDP ratio is high when an asset price bubble exists. Our definition of credit is

$$credit_t = \pi \left[\phi q_t \left(1 - \delta\left(u_t\right)\right) K_t + \mathbf{1}_{\{z_t = b\}} \tilde{p}_t M \right]. \tag{28}$$

This is the amount of funds that flows from savers to investors in the aggregate. In Figure 6, we plot it relative to aggregate output. The clear and robust pattern is that the credit-to-GDP ratio is high when bubbles exist. The reason is obvious; more funds flow from savers to investors if investors can sell bubbly assets at a positive price.

Strictly speaking, our measure of credit is the total asset sales in the economy. But we emphasize that this is just a matter of interpretation. Indeed, we can justify our definition of credit with a slight modification to the model. There, investors borrow funds from savers but need collateral for that. Collateralizability, however, differs across asset classes. Namely, only a fraction ϕ of capital is collateralizable, but for bubbly assets, everything is. Investors as a group obtain credit from savers.

Figure 7 shows the observables (left panel GDP growth and right panel credit-to-GDP, where credit corresponds to total credit to the private non-financial sector). For convenience, the red line shows the 10-year rolling-window average of GDP growth. It is not difficult to see the slowdown in GDP growth in the sample, going from 0.7% (2.8 % in annual terms) in the 1990s, to 0.87% (3.5 %) in 2005, to less than 0.4% (or 1.6 %) after the Great Recession. Credit-to-GDP displayed some moderate variation during the period 1984-2000.

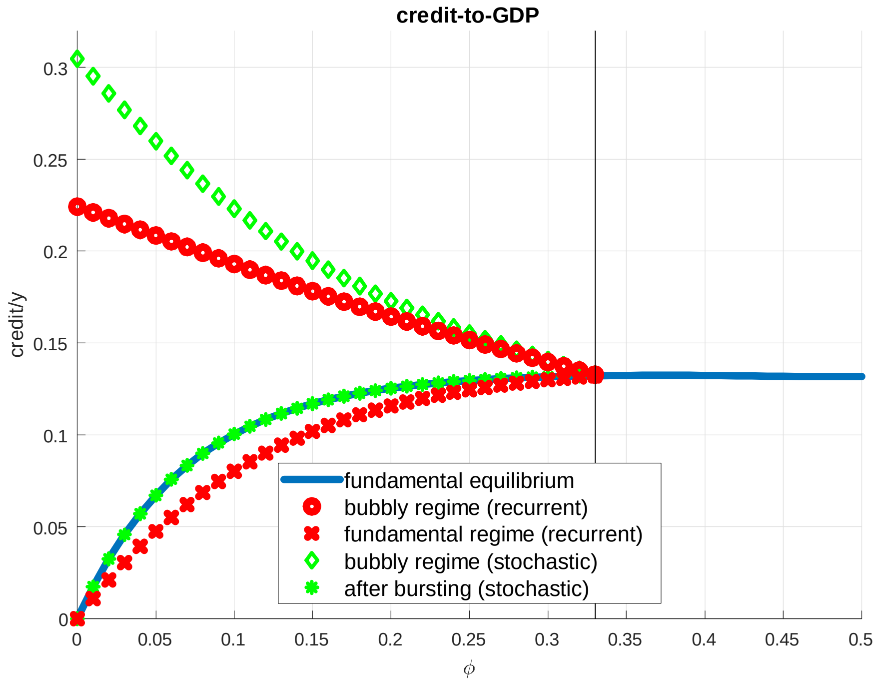

Figure 6: Liquidity and Credit-to-GDP in Recurrent-Bubble Model

During the years leading up to the Great Recession, GDP growth was high, averaging 3% per year (black circles in Figure 7) at the same time that credit expanded aggressively, reaching an all-time high in 2008 (about 5 times larger than in the previous peak in the late 1990s). We observe the opposite during the post-crisis years; lackluster growth of 1.6% (black diamonds) and a sharp contraction in credit going back to its 1990s level. We exploit this connection between growth and credit to uncover the presence of bubbles in the data. That is, our objective is to capture Mishkin (2011)'s "credit-driven bubbles" rather than irrational exuberance bubbles. In Mishkin's view, the former type of bubbles were prevalent in the Great Recession while the latter class was prevalent in the 1990 – Jorda, Schularick, and Taylor (2015) find empirical support to Mishkin's claim.

We estimate the model using Bayesian methods (Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide (2016)) and Kim's filter (Kim and Nelson (1999)), assuming that the economy is in the bubbly equilibrium.[11] We impose the condition that the productivity- and preference-shock processes have an AR(1) structure and estimate the persistence, ρ_i, and volatility, σ_i, of these stochastic processes ($i =$\{productivity (a), preference (b)\}). We impose standard beta and inverse-gamma priors for these parameters.

[11] Our model follows within the class of MS-DSGE models discussed in Farmer, Waggoner, and Zha (2009). We find a fundamental minimum-state-variable equilibrium. The absence of endogenous state variables greatly simplifies the solution method, as otherwise we would have to rely on the methods in Farmer, Waggoner, and Zha (2011).

The two shocks have distinct impacts on the observables. Namely, the productivity shock raises GDP growth temporarily, while the preference shock raises the credit-to-GDP ratio temporarily. However, they have a milder impact on other variables. Responses are modestly regime-dependent, slightly larger in the bubbly regime than in the fundamental regime. Both shocks have positive impacts on investment and accelerate capital accumulation temporarily (see the appendix for detail).

Except for the liquidity parameter, ϕ, all other parameter values are those in Table 1. Recall that the liquidity parameter was a free parameter in the previous sections, since our objective was to analyze its impact on different versions of our model. In this section, we choose $\phi = 0.19$, which is in line with Kiyotaki and Moore (Forthcoming).

Our identification of the regimes exploits the implications shown in Figures 1 and 6 that the bubbly regime is characterized by both higher credit-to-GDP and higher economic growth. Our calibrated model already has sharp predictions for the means of those variables in each regime. But these model-implied means may not match their data counterparts because they are not included in the calibration targets. Moreover, we do not want to impose their empirical counterparts directly from calibration because that exercise needs to take an a priori stance on when the bubbles existed in the economy. Rather, we implement our identification strategy by estimating the average growth rate and credit-to-GDP during the fundamental and bubbly regimes. That is, we estimate the average growth rate in the data if in the fundamental regime, $\mu_{g,f}$, as the sum of the model's implied growth rate in the fundamental regime, $\mu_{g,f}^{m}$, and an offseting constant, $\bar{\mu}_{g,f}$ ($\mu_{g,f} = \mu_{g,f}^{m} + \bar{\mu}_{g,f}$). Here, the constant makes up for the difference between the model and data growth rates. A similar strategy is imposed on growth in the bubbly regime and credit-to-GDP in the two regimes. In the appendix and in a previous working paper version, we show that our findings are robust to alternative calibration and identification strategies such as 1) using ϕ to match the means of GDP growth and the credit-to-GDP ratio in and out of the Great Recession with the caveat that we impose the dates when the bubble exists; 2) a longer sample; 3) a third regime featuring high growth and high credit-to-GDP driven by non-bubble forces; and 4) GDP growth and the consumption-to-investment ratio as observables.

We use fairly agnostic normal priors for the means of GDP growth and credit-to-GDP in the fundamental and bubbly regimes, $\{\mu_{g,f}, \mu_{credit/y,f}, \mu_{g,b}, \mu_{credit/y,b}\}$, respectively. Table 2 presents both the priors and posteriors (mode and 90% credible bands) from the estimation. The priors and posteriors are different, which points to the informativeness of the data. Importantly, the posterior modes indicate that GDP growth and credit-to-GDP are higher during the bubbly regime than during the fundamental regime.[12] For example, the estimated average GDP growth is about 40 basis points higher in the presence of bubbles than in periods without them. In terms of the structural shocks, we observe that the preference disturbance is volatile, but lacks persistence

[12] The offseting constants are estimated to be negative, which indicates that the regime-dependent means implied by the model are higher than in the data.

while the productivity shock is persistent but relatively smooth.[13] Interestingly, the estimated persistence of productivity is significantly lower than the typical number in the literature (≈ 0.95), which is a consequence of the persistence introduced by the regime switching.

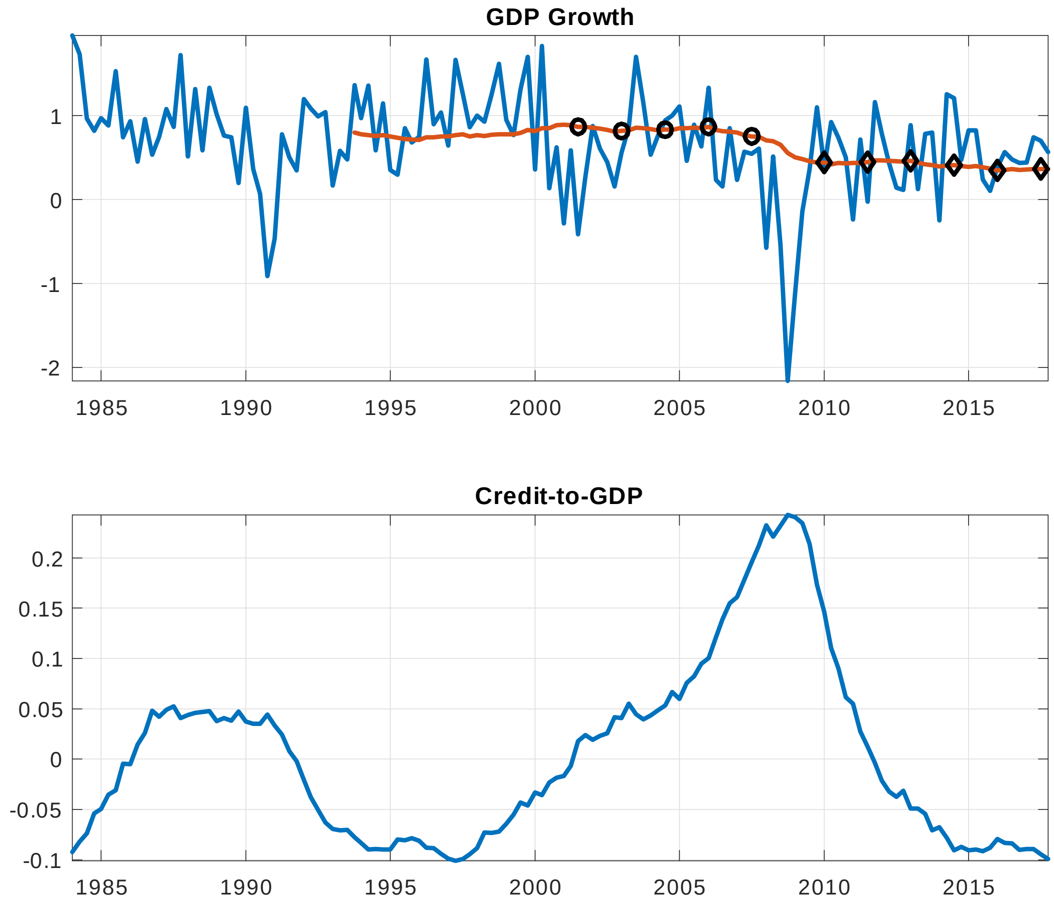

Figure 7: GDP Growth and Credit-to-GDP Ratio in Data

6.1 Results

Figure 8 presents the estimated probability of the economy being in the bubbly regime. It shows that the economy spent in the fundamental regime prior to the 2000s. This means that during the first 15 years of the sample, growth was driven by exogenous productivity shocks (upper panel in Figure 9), not a surprise given the moderate credit-to-GDP ratio in the data.

The economy starts the 2000s in the fundamental regime, but as credit expands rapidly, the probability of being in the bubbly regime rises. By mid-2005, the bubble is becoming more likely,

[13] We tried alternative means and standard deviations for the priors. Our results are robust to these variations. This should not be surprising given how tightly estimated the parameters are.

Parameter	Prior	Posterior
$\mu_{g,f}$	$N(0.5, 0.1)$	0.14 [0.13,0.15]
$\mu_{credit/y,f}$	$N(-0.05, 0.1)$	-0.037 [-0.039,-0.035]
$\mu_{g,b}$	$N(0.75, 0.1)$	0.56 [0.53,0.59]
$\mu_{credit/y,b}$	$N(0.15, 0.1)$	0.047 [0.045,0.05]
ρ_b	$B(0.5, 0.1)$	0.30 [0.29,0.32]
σ_b	$IG(6, 1)$	0.14 [0.13,0.15]
ρ_a	$B(0.5, 0.1)$	0.71 [0.68,0.75]
σ_a	$IG(6, 1)$	0.06 [0.057,0.063]

Table 2: Estimated Parameters

with a smoothed probability above 50%. Between 2007 and early 2009, our exercise reveals that the bubble was in full swing. Importantly, growth is bubble-driven in this period, which is an interesting contrast to the productivity-driven growth in the 1990s; note that productivity shocks are generally smaller in the early and mid-2000s than in the 1990s (Figure 9). At its peak, credit in the data is explained by a combination of bubbles and a high productivity shock. The bubble disappears in the early 2010s.

During the initial phase of the Great Recession, credit is in correction territory but high compared to the 1990s. As a consequence, our approach identifies this stage of the crisis as the result of a sharp decline in investment demand due to an exogenous shock to preferences. But as the contraction in credit continued and the economy grew at lackluster rates, the fundamental regime becomes more likely to the point where it is the prevalent regime since 2011. It is worth noting that our estimate of the bubbly episode lasts longer than other researchers have found (Jorda, Schularick, and Taylor (2015)). This is due to the evolution of aggregate credit, peaking at the end of 2008 and slowly retrenching afterward, the latter of which Ivashina and Scharfstein (2010) attribute to the extensive use of existing lines of credit during 2009 and 2010. Ideally, we would use newly issued credit rather than total credit to better capture the narrative behind the crisis. However, to the best of our knowledge, such data are not available at the frequency and length required for our purpose.

Interestingly, the timing of the regime switches in our model looks different from estimates in other regime-switching models advanced in the literature. For example, Sims and Zha (2006) fit U.S. data to a regime-switching VAR with drifting coefficients and variances. They report the existence of four distinct regimes: the Greenspan state prevailing during the 1990s and early 2000s; the second most common regime emerges in the early 1960s and parts of the 1970s; the last two regimes correspond to sporadic events such as 9/11. Our regimes are unlike those estimated to account for the Great Moderation, with a high volatility regime prior to 1984 and a calmer one post-

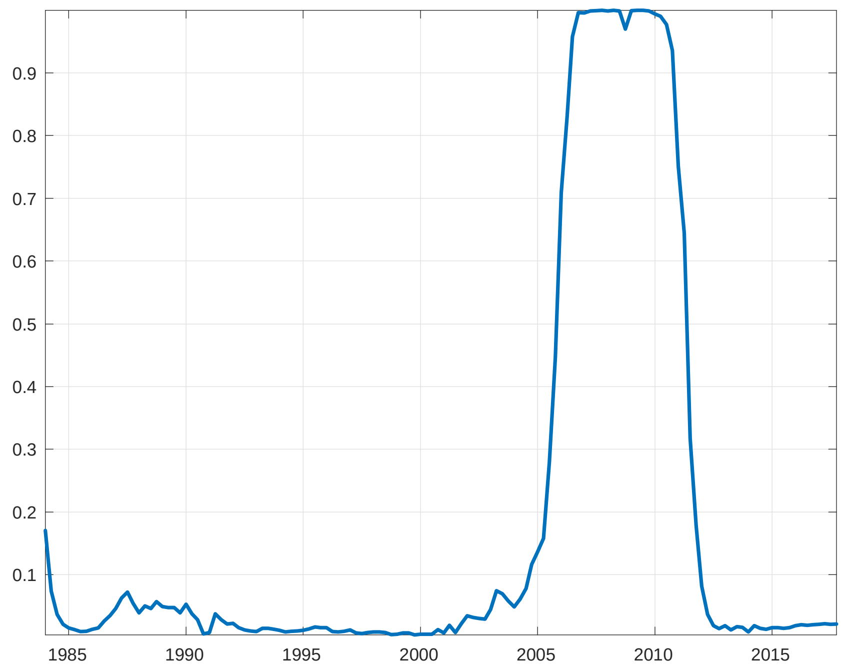

Figure 8: Probability of Bubbly Regime

1984 (Stock and Watson (2002)). Finally, our bubbly regime bears little resemblance to recession regimes. (See Hamilton (2016) for an extensive review of regime switching in macroeconomics.)

Next, we expand our discussion of the estimated structural shocks in our model (Figure 9). We observe that business cycles in the fundamental regime were mostly driven by innovations in exogenous productivity. Clearly, the 1991 and 2001 recessions are the result of drops in productivity. Positive productivity shocks were behind the expansion in the 1990s. For most of the sample, demand shifters are estimated to be favorable for investment and growth. As we enter the pre-Great Recession years (2006 - 2007), there is a significant increase in productivity. Clearly, credit growth was so robust in the years leading up to the recession that the only way for the model to explain the data is through a combination of the switch to the bubbly regime and a spike in productivity. Interestingly, the run-up in productivity is consistent with the utilization-adjusted TFP reported by Fernald (2015). As credit loses steam and the economy enters the recession, productivity drops dramatically. It is the preference shock that causes the slowdown in GDP growth around the demise of Lehman Brothers. Although exogenous productivity has been high in recent years, it is about half the size of that during the 1990s. As a result, the lackluster recovery post-Great Recession is a combination of weak exogenous productivity and the return of the economy to the fundamental regime.

According to our analysis, it is very likely that there was a bubbly episode in the period 2005

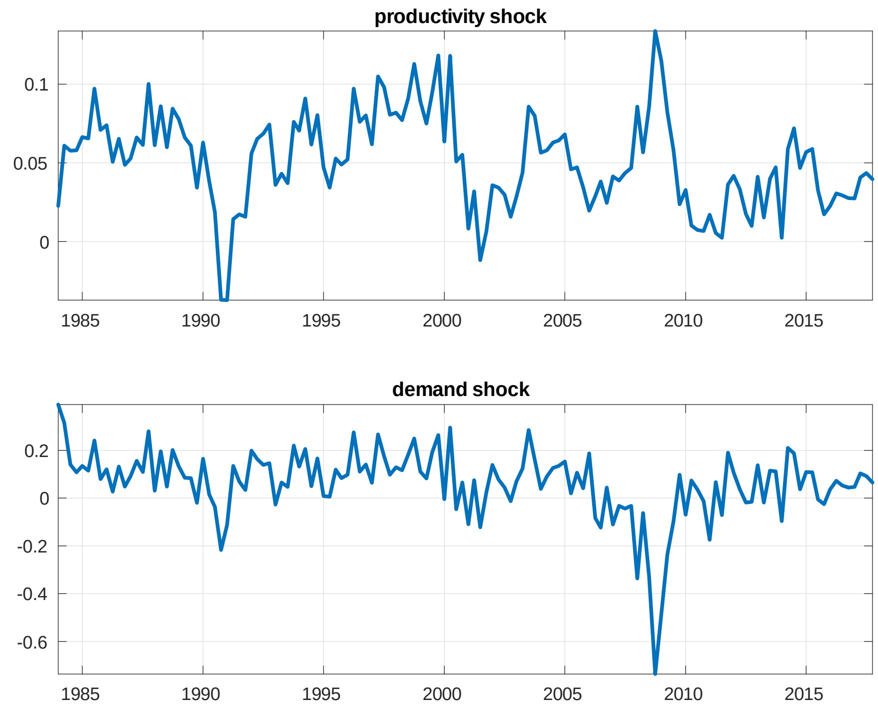

Figure 9: Estimated Demand and Productivity Shocks

- 2011. Its contribution to U.S. economic growth can be assessed by counterfactual simulations. First, we simulate the economy with the same structural shocks but imposing the condition that the bubble in the 2000s did not materialize; i.e., the probability of the economy being in the bubbly regime was artificially set at zero throughout the sample. Yet, we assume that the economy is in the recurrent-bubble equilibrium; it is only the realization of the regime that we change. The red dashed line in Figure 10 shows the growth rate of the economy under this counterfactual scenario; the blue solid line shows the growth rate in the benchmark scenario. Prior to 2005, the two lines are virtually identical because the bubble was very unlikely. However, as the bubble becomes more likely, growth under the counterfactual scenario becomes weaker than in the actual path. The difference between these two cases is more apparent when we look around the Great Recession. At points in time, the difference is as large as 50 basis points. The accumulated impact is staggering; had the bubble not materialized circa 2005, the economy's trend would have been 10 percentage points below the actual trend by 2011, and this gap is permanent. In this sense, the impact of the bubble in the 2000s is both positive and large.

But as we emphasized in the previous section, no realization of bubbles by chance is different from no chance of realization. We simulate the latter case (fundamental equilibrium), and plot the counterfactual GDP growth in the black dotted line in Figure 10.[14] Clearly, the economy

[14] We continue to use the estimated structural shocks for this exercise.

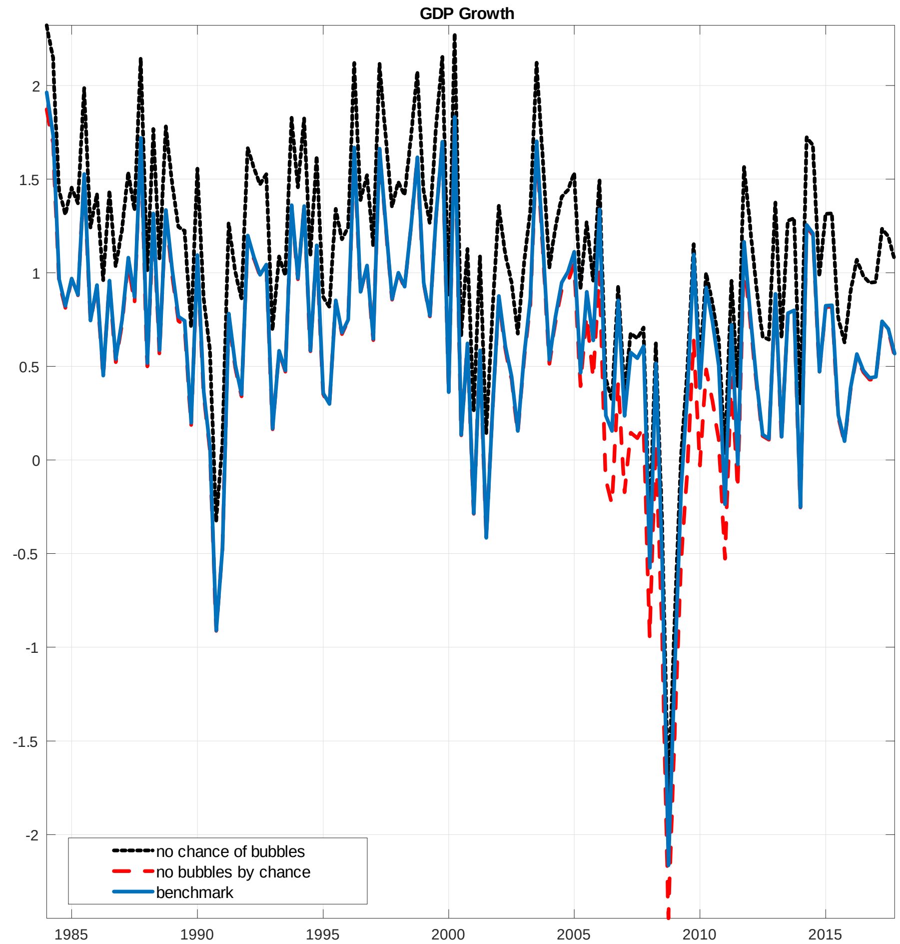

Figure 10: GDP under Counterfactual Scenarios

with no chance of bubbles would have grown at a faster rate than in the benchmark case or in the economy without bubbles by chance (on average, 40 basis points higher per quarter). This is because of (the absence of) the crowding out effect of future bubbles; had they not expected bubbles to re-emerge, people would have consumed less, worked more, and invested more, all of which would have contributed to growth.

7 Conclusions

We have advanced a model of recurrent bubbles, liquidity, and endogenous productivity. Unlike the previous work in the literature (Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2012), Miao, Wang, and Xu (2015), and Hirano and Yanagawa (2017)), we introduce recurrent bubbles with entire collapse and reappearance in an infinite horizon model. A novel crowding out effect of asset price bubbles emerges; that is, expectations about future bubbles increase both consumption and leisure, decrease both investment and labor supply, and hence slow down economic growth. This crowding out effect exists not only in the bubbleless periods but also in the bubbly periods. In contrast, the crowding in effect of realized bubbles exists only in the bubbly period. Hence, no bubbles by chance is different from no chance of bubbles. The former case suffers from the crowding out effect of future bubbles to the extent that people expect them, while the latter case is free from it. Economic growth in the former case is generally slower than in the latter case.

The impact of recurrent bubbles on economic growth and welfare depends on a balance between the crowding in effect of realized bubbles and the crowding out effect of future bubbles, which in turn depend on economic fundamentals. If the financial market is under-developed, the crowding in effect dominates, and recurrent bubbles raise average economic growth and welfare compared to those of the case where asset price bubbles never arise. If the financial market is developed, the benefit of the crowding in effect is weak and may be dominated by the crowding out effect. The optimal frequency of bubbles exists, and it too depends on economic fundamentals. The economy with a severely under-developed financial system generally prefers high-frequency bubbles. But as the financial system gradually develops, the economy starts to prefer lower-frequency bubbles partly because the crowding out effect is weaker (stronger) if bubbles are less (more) frequent.

These results have new policy implications.[15] Our paper suggests that leaning against the bubble policy can be justified in advanced economies because their financial markets are generally developed. Such an economy will not benefit from the crowding in effect of realized bubbles but should worry more about the crowding out effect of future bubbles. Complete elimination of bubbles might not be easy in practice, but if the frequency of bubbles can be reduced, that would already be a step in the right direction. On the other hand, in developing economies with under-developed financial markets, leaning with bubble policy might promote growth and welfare. Increasing the frequency of bubbles might not be a bad idea either. But the welfare-maximizing frequency of bubbles is country specific.

We have examined U.S. economic growth performance and its causes through the lens of our recurrent-bubble model. According to our estimates, booms in the 1990s and the 2000s were driven by distinct forces. The boom in the 1990s was driven by improvement in exogenous productivity, but the boom in the 2000s was driven by the emergence of an asset price bubble. The lackluster recovery post-Great Recession was due to a combination of the bursting of the asset price bubble and unlucky draws of exogenous shocks. The counterfactual simulation reveals that the asset price bubble in the 2000s raised U.S. GDP. But the positive contribution of realized bubbles per se does not necessarily mean that bubbles are unconditionally desirable. Indeed, another counterfactual

[15] See Gali (2014), Hirano, Inaba, and Yanagawa (2015), and Allen, Barlevy, and Gale (2017) for a discussion of asset price bubbles and government policy.

simulation reveals that U.S. economic growth performance would be better in the hypothetical, but theoretically possible, equilibrium in which bubbles never arise. In other words, actual realization of a bubble is better than no realization of bubbles by chance, but it is even better for the U.S. economy to move to another equilibrium in which there is no chance of bubbles at all and people recognize this impossibility.

References

Aghion, P., P. Howitt, and D. Mayer-Foulkes (2005): "The Effect of Financial Development on Convergence: Theory and Evidence," *The Quarterly Journal of Economics*, 120(1), 173.

Allen, F., G. Barlevy, and D. Gale (2017): "On Interest Rate Policy and Asset Bubbles," Unpublished Manuscript.

Arrow, K. J. (1962): "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, 29(3), pp. 155–173.

Bassett, W., A. Daigle, R. Edge, and G. Kara (2015): "Credit-to-GDP Trends and Gaps by Lender- and Credit-Type," FEDS Notes. December 3, 2015, Washington: Board of Governors of the Federal Reserve System.

Bernanke, B. (2018): "The Real Effects of the Financial Crisis," in *BPEA Conference Draft, Fall.*

Bewley, T. (1980): "The optimum quantity of money," in *Models of Monetary Economies*, ed. by J. Kareken, and N. Wallace, pp. 169–210. Federal Reserve Bank of Minneapolis.

Biswas, S., A. Hanson, and T. Phan (2018): "Bubbly Recessions," Working Paper No. 18-05, Federal Reserve Bank of Richmond.

Blanchard, O., E. Cerutti, and L. Summers (2014): "Inflation and Activity – Two Explorations, and Their Monetary Policy Implications," Discussion paper, ECB Forum on Central Banking.

Brunnermeier, M. K., and M. Oehmke (2013): "Bubbles, Financial Crises, and Systemic Risk," in *Handbook of the Economics of Finance*, ed. by G. M. Constantinides, M. Harris, and R. M. Stulz, vol. 2, chap. 18, pp. 1221 – 1288. Elsevier.

Brunnermeier, M. K., and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104(2), 379–421.

Cerra, V., and S. C. Saxena (2008): "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review*, 98(1), 439–57.

Comin, D., and M. Gertler (2006): "Medium-Term Business Cycles," *American Economic Review*, 96, 523–551.

Diba, B. T., and H. I. Grossman (1988): "The Theory of Rational Bubbles in Stock Prices," *The Economic Journal*, 98(392), 746–754.

Farhi, E., and J. Tirole (2012): "Bubbly Liquidity," *The Review of Economic Studies*, 79(2), 678.

Farmer, R. E., D. F. Waggoner, and T. Zha (2009): "Understanding Markov-switching rational expectations models," *Journal of Economic Theory*, 144(5), 1849 – 1867.

——— (2011): "Minimal state variable solutions to Markov-switching rational expectations models," *Journal of Economic Dynamics and Control*, 35(12), 2150 – 2166, Frontiers in Structural Macroeconomic Modeling.

Fernald, J. G. (2015): "Productivity and Potential Output before, during, and after the Great Recession," *NBER Macroeconomics Annual*, 29(1), 1–51.

Fernandez-Villaverde, J., J. Rubio-Ramirez, and F. Schorfheide (2016): "Solution and Estimation Methods for DSGE Models," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2, chap. 9, pp. 527 – 724. Elsevier.

Gali, J. (2014): "Monetary Policy and Rational Asset Price Bubbles," *American Economic Review*, 104(3), 721–52.

——— (2016): "Insider-outsider labor markets, hysteresis and monetary policy," Economics Working Papers 1506, Department of Economics and Business, Universitat Pompeu Fabra.

Gertler, M., and N. Kiyotaki (2015): "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, 105(7), 2011–43.

Guerron-Quintana, P. A., and R. Jinnai (Forthcoming): "Financial Frictions, Trends, and the Great Recesssion," *Quantitative Economics*.

Hamilton, J. D. (2016): "Macroeconomic Regimes and Regime Shifts," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2 of *Handbook of Macroeconomics*, chap. 3, pp. 163 – 201. Elsevier.

He, Z., and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 103(2), 732–70.

Hirano, T., M. Inaba, and N. Yanagawa (2015): "Asset bubbles and bailouts," *Journal of Monetary Economics*, 76, S71 – S89, Supplement Issue: November 7-8, 2014 Research Conference on Asset Price Fluctuations and Economic Policy.

Hirano, T., and N. Yanagawa (2017): "Asset Bubbles, Endogenous Growth, and Financial Frictions," *The Review of Economic Studies*, 84(1), 406–443.

Ivashina, V., and D. Scharfstein (2010): "Bank lending during the financial crisis of 2008," *Journal of Financial Economics*, 97(3), 319 – 338.

Jorda, O., M. Schularick, and A. M. Taylor (2015): "Leveraged bubbles," *Journal of Monetary Economics*, 76, S1 – S20, Supplement Issue: November 7-8, 2014 Research Conference on Asset Price Fluctuations and Economic Policy.

Kamihigashi, T. (2011): "Recurrent bubbles," *The Japanese Economic Review*, 62(1), 27–62.

Kim, C.-J., and C. R. Nelson (1999): *State-Space Models with Regime Switching*. The MIT Press.

Kindleberger, C. P. (2001): *Manias, Panics and Crashes*. Wiley, 4th edn.

Kiyotaki, N., and J. Moore (Forthcoming): "Liquidity, Business Cycles, and Monetary Policy," *Journal of Political Economy*.

Kocherlakota, N. (2009): "Bursting Bubbles: Consequences and Cures," Unpublished Manuscript, University of Minnesota.

Kocherlakota, N. R. (1992): "Bubbles and constraints on debt accumulation," *Journal of Economic Theory*, 57(1), 245 – 256.

Martin, A., and J. Ventura (2012): "Economic Growth with Bubbles," *American Economic Review*, 102(6), 3033–58.

Miao, J., P. Wang, and Z. Xu (2015): "A Bayesian dynamic stochastic general equilibrium model of stock market bubbles and business cycles," *Quantitative Economics*, 6(3), 599–635.

Mishkin, F. S. (2011): "Monetary Policy Strategy: Lessons from the Crisis," Working Paper 16755, National Bureau of Economic Research.

Ranciere, R., A. Tornell, and F. Westermann (2008): "Systemic Crises and Growth," *The Quarterly Journal of Economics*, 123(1), 359–406.

Romer, P. M. (1986): "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94(5), pp. 1002–1037.

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66(6), 467–482.

Scheinkman, J. A., and L. Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1), 23–45.

Schmitt-Grohe, S., and M. Uribe (2005): "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model," Working Paper 11854, National Bureau of Economic Research.

Shell, K., M. Sidrauski, and J. E. Stiglitz (1969): "Capital Gains, Income, and Saving," *The Review of Economic Studies*, 36(1), 15–26.

Sheshinski, E. (1967): "Optimal accumulation with learning by doing," in *Essays on the Theory of Optimal Economic Growth*, ed. by K. Shell, pp. 31–52. MIT Press, Cambridge, MA.

Shi, S. (2015): "Liquidity, assets and business cycles," *Journal of Monetary Economics*, 70, 116 – 132.

Sims, C. A., and T. Zha (2006): "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 96(1), 54–81.

Stock, J., and M. Watson (2002): "Has the Business Cycle Changed and Why?," *NBER Macroeconomics Annual*, pp. 159–230.

Tirole, J. (1982): "On the Possibility of Speculation under Rational Expectations," *Econometrica*, 50(5), 1163–1181.

——— (1985): "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6), 1499–1528.

Townsend, R. M. (1980): "Models of Money with Spatially Separated Agents," in *Models of Monetary Economies*, ed. by J. Kareken, and N. Wallace, pp. 265–303. Federal Reserve Bank of Minneapolis.

Weil, P. (1987): "Confidence and the Real Value of Money in an Overlapping Generations Economy," *The Quarterly Journal of Economics*, 102(1), 1.

Woodford, M. (1990): "Public Debt as Private Liquidity," *The American Economic Review*, 80(2), 382–388.

8 Appendix

NOT FOR PUBLICATION

8.1 Permanent Fundamental

8.1.1 Without Binding Inequality Constraints

The household's problem is

$$\max E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t}{e^{b_t}} \left(\pi \frac{[c_t^i]^{1-\rho}}{1-\rho} + (1-\pi) \frac{[c_t^s (1-l_t)^{\eta}]^{1-\rho}}{1-\rho} \right) \right]$$

subject to

$$\pi c_t^i + (1-\pi) c_t^s + n_{t+1} - (1-\delta(u_t)) n_t = u_t r_t n_t + w_t (1-\pi) l_t.$$

The equilibrium conditions are summarized as follows:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t ((1-\pi) l_t)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = (c_t^s)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$\delta'(u_t) = r_t,$$

$$1 = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\rho} (u_{t+1} r_{t+1} + 1 - \delta(u_{t+1})) \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi) l_t},$$

and

$$\pi c_t^i + (1-\pi) c_t^s + K_{t+1} - (1-\delta(u_t)) K_t = Y_t$$

for all t.

Detrend variables by K_t:

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} ((1-\pi) l_t)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = (\hat{c}_t^s)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$\delta'(u_t) = r_t,$$

$$1 = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i} \frac{1}{g_t} \right)^{\rho} \left(u_{t+1} r_{t+1} + 1 - \delta\left(u_{t+1}\right) \right) \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

and

$$\pi \hat{c}_t^i + (1-\pi) \hat{c}_t^s + g_t - (1 - \delta(u_t)) = \hat{Y}_t.$$

8.1.2 With Binding Inequality Constraints

The household's problem is

$$\max E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t}{e^{b_t}} \left(\pi \frac{\left[c_t^i\right]^{1-\rho}}{1-\rho} + (1-\pi) \frac{\left[c_t^s (1-l_t)^{\eta}\right]^{1-\rho}}{1-\rho} \right) \right]$$

subject to

$$\pi c_t^i + (1-\pi) c_t^s + q_t n_{t+1} = \left[u_t r_t + (1-\delta(u_t)) q_t + \pi \lambda_t \left(u_t r_t + \phi q_t (1 - \delta(u_t)) \right) \right] n_t + (1-\pi) w_t l_t$$

A competitive equilibrium is defined as a sequence of prices, w_t, r_t, and q_t, and quantities, Y_t, i_t, K_{t+1}, c_t^i, c_t^s, l_t, and u_t, that satisfy the following conditions:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t \left((1-\pi) l_t \right)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t \left(r_t - \phi q_t \delta'(u_t) \right) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{c_t^i}{c_{t+1}^i} \right)^{\rho} \left(u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} \left(u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})) \right) \right) \right],$$

$$r_t = \alpha \frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha) \frac{Y_t}{(1-\pi) l_t},$$

$$Y_t = \pi c_t^i + (1-\pi) c_t^s + \pi \frac{\left[u_t r_t + \phi q_t (1-\delta(u_t)) \right] K_t}{1 - \phi q_t},$$

and

$$K_{t+1} = (1-\delta(u_t)) K_t + \pi \frac{[u_t r_t + \phi q_t (1-\delta(u_t))] K_t}{1-\phi q_t}$$

for all t.

Since the model displays endogenous productivity, it is necessary to detrend it before we solve it numerically. Dividing quantities by K_t, we obtain the following equations.

$$\hat{Y}_t = \bar{A} e^{a_t} u_t^{\alpha} ((1-\pi) l_t)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = \left(\hat{c}_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t \left[\frac{\beta}{e^{b_{t+1}-b_t}} \left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i} \frac{1}{g_t} \right)^{\rho} (u_{t+1} r_{t+1} + (1-\delta(u_{t+1})) q_{t+1} + \pi \lambda_{t+1} (u_{t+1} r_{t+1} + \phi q_{t+1} (1-\delta(u_{t+1})))) \right],$$

$$r_t = \alpha \frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha) \frac{\hat{Y}_t}{(1-\pi) l_t},$$

$$\hat{Y}_t = \pi \hat{c}_t^i + (1-\pi) \hat{c}_t^s + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t))}{1-\phi q_t},$$

and

$$g_t = 1 - \delta(u_t) + \pi \frac{u_t r_t + \phi q_t (1-\delta(u_t))}{1-\phi q_t}$$

for all t, where hat variables denote the original variable divided by K_t, i.e., $\hat{Y}_t \equiv Y_t / K_t$ and so on, and $g_t \equiv K_{t+1}/K_t$.

8.2 Recurrent-Bubble Model

Competitive equilibrium is summarized by the following equations:

$$Y_t = \bar{A} e^{a_t} u_t^{\alpha} K_t ((1-\pi) l_t)^{1-\alpha},$$

$$\left(c_t^i\right)^{-\rho} = \left(c_t^s\right)^{-\rho} (1-l_t)^{\eta(1-\rho)},$$

$$\eta \frac{c_t^s}{1-l_t} = w_t,$$

$$r_t - \delta'(u_t) q_t + \pi \lambda_t (r_t - \phi q_t \delta'(u_t)) = 0,$$

$$q_t = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho}\left(u_{t+1}r_{t+1} + (1-\delta(u_{t+1}))q_{t+1} + \pi\lambda_{t+1}\left(u_{t+1}r_{t+1} + \phi q_{t+1}(1-\delta(u_{t+1}))\right)\right)\right],$$

$$\mathbf{1}_{\{z_t=b\}}\tilde{p}_t = \mathbf{1}_{\{z_t=b\}}E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{c_t^i}{c_{t+1}^i}\right)^{\rho}(1+\pi\lambda_{t+1})\tilde{p}_{t+1}\mathbf{1}_{\{z_{t+1}=b\}}\right],$$

$$r_t = \alpha\frac{Y_t}{u_t K_t},$$

$$w_t = (1-\alpha)\frac{Y_t}{(1-\pi)l_t},$$

$$Y_t = \pi c_t^i + (1-\pi)c_t^s + \pi\frac{[u_t r_t + \phi q_t(1-\delta(u_t))]K_t + \tilde{p}_t\mathbf{1}_{\{z_t=b\}}M}{1-\phi q_t},$$

$$K_{t+1} = (1-\delta(u_t))K_t + \pi\frac{[u_t r_t + \phi q_t(1-\delta(u_t))]K_t + \tilde{p}_t\mathbf{1}_{\{z_t=b\}}M}{1-\phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\phi q_t}.$$

Dividing variables by K_t, we find

$$\hat{Y}_t = \bar{A}e^{a_t}u_t^{\alpha}\left((1-\pi)l_t\right)^{1-\alpha},$$

$$\left(\hat{c}_t^i\right)^{-\rho} = \left(\hat{c}_t^s\right)^{-\rho}(1-l_t)^{\eta(1-\rho)},$$

$$\eta\frac{\hat{c}_t^s}{1-l_t} = \hat{w}_t,$$

$$r_t - \delta'(u_t)q_t + \pi\lambda_t(r_t - \phi q_t\delta'(u_t)) = 0,$$

$$q_t = E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i}\frac{1}{g_t}\right)^{\rho}\left(u_{t+1}r_{t+1} + (1-\delta(u_{t+1}))q_{t+1} + \pi\lambda_{t+1}\left(u_{t+1}r_{t+1} + \phi q_{t+1}(1-\delta(u_{t+1}))\right)\right)\right],$$

$$m_t = \mathbf{1}_{\{z_t=b\}}E_t\left[\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}_t^i}{\hat{c}_{t+1}^i}\frac{1}{g_t}\right)^{\rho}(1+\pi\lambda_{t+1})m_{t+1}g_t\right],$$

$$r_t = \alpha\frac{\hat{Y}_t}{u_t},$$

$$\hat{w}_t = (1-\alpha)\frac{\hat{Y}_t}{(1-\pi)l_t},$$

$$\hat{Y}_t = \pi\hat{c}_t^i + (1-\pi)\hat{c}_t^s + \pi\frac{u_t r_t + \phi q_t(1-\delta(u_t)) + m_t}{1-\phi q_t},$$

$$g_t = 1-\delta(u_t) + \pi\frac{u_t r_t + \phi q_t(1-\delta(u_t)) + m_t}{1-\phi q_t},$$

and

$$\lambda_t = \frac{q_t - 1}{1-\phi q_t}$$

where $m_t \equiv \tilde{p}_t \mathbf{1}_{\{z_t=b\}} M/K_t$.

It is convenient to make the dependence on the regime explicit:

$$\hat{Y}_{f,t} = \bar{A}e^{a_t} (u_{f,t})^{\alpha} ((1-\pi) l_{f,t})^{1-\alpha}, \tag{29}$$

$$\hat{Y}_{b,t} = \bar{A}e^{a_t} (u_{b,t})^{\alpha} ((1-\pi) l_{b,t})^{1-\alpha}, \tag{30}$$

$$\left(\hat{c}^i_{f,t}\right)^{-\rho} = \left(\hat{c}^s_{f,t}\right)^{-\rho} (1-l_{f,t})^{\eta(1-\rho)}, \tag{31}$$

$$\left(\hat{c}^i_{b,t}\right)^{-\rho} = \left(\hat{c}^s_{b,t}\right)^{-\rho} (1-l_{b,t})^{\eta(1-\rho)}, \tag{32}$$

$$\eta \frac{\hat{c}^s_{f,t}}{1-l_{f,t}} = \hat{w}_{f,t}, \tag{33}$$

$$\eta \frac{\hat{c}^s_{b,t}}{1-l_{b,t}} = \hat{w}_{b,t}, \tag{34}$$

$$r_{f,t} - \delta'(u_{f,t}) q_{f,t} + \pi\lambda_{f,t} (r_{f,t} - \phi q_{f,t}\delta'(u_{f,t})) = 0, \tag{35}$$

$$r_{b,t} - \delta'(u_{b,t}) q_{b,t} + \pi\lambda_{b,t} (r_{b,t} - \phi q_{b,t}\delta'(u_{b,t})) = 0, \tag{36}$$

$$\begin{aligned} q_{f,t} &= E_t\left[\begin{array}{c} (1-\sigma_f)\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{f,t}}{\hat{c}^i_{f,t+1}}\frac{1}{g_{f,t}}\right)^{\rho} \\ (u_{f,t+1}r_{f,t+1} + (1-\delta(u_{f,t+1}))q_{f,t+1} + \pi\lambda_{f,t+1}(u_{f,t+1}r_{f,t+1} + \phi q_{f,t+1}(1-\delta(u_{f,t+1})))) \end{array}\right] \\ &+E_t\left[\begin{array}{c} \sigma_f\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{f,t}}{\hat{c}^i_{b,t+1}}\frac{1}{g_{f,t}}\right)^{\rho} \\ (u_{b,t+1}r_{b,t+1} + (1-\delta(u_{b,t+1}))q_{b,t+1} + \pi\lambda_{b,t+1}(u_{b,t+1}r_{b,t+1} + \phi q_{b,t+1}(1-\delta(u_{b,t+1})))) \end{array}\right], \end{aligned} \tag{37}$$

$$\begin{aligned} q_{b,t} &= E_t\left[\begin{array}{c} (1-\sigma_b)\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{b,t+1}}\frac{1}{g_{b,t}}\right)^{\rho} \\ (u_{b,t+1}r_{b,t+1} + (1-\delta(u_{b,t+1}))q_{b,t+1} + \pi\lambda_{b,t+1}(u_{b,t+1}r_{b,t+1} + \phi q_{b,t+1}(1-\delta(u_{b,t+1})))) \end{array}\right] \\ &+E_t\left[\begin{array}{c} \sigma_b\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{f,t+1}}\frac{1}{g_{b,t}}\right)^{\rho} \\ (u_{f,t+1}r_{f,t+1} + (1-\delta(u_{f,t+1}))q_{f,t+1} + \pi\lambda_{f,t+1}(u_{f,t+1}r_{f,t+1} + \phi q_{f,t+1}(1-\delta(u_{f,t+1})))) \end{array}\right], \end{aligned} \tag{38}$$

$$m_{f,t} = 0, \tag{39}$$

$$\begin{aligned} m_{b,t} &= E_t\left[(1-\sigma_b)\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{b,t+1}}\frac{1}{g_{b,t}}\right)^{\rho}(1+\pi\lambda_{b,t+1})m_{b,t+1}g_{b,t}\right] \\ &+E_t\left[\sigma_b\frac{\beta}{e^{b_{t+1}-b_t}}\left(\frac{\hat{c}^i_{b,t}}{\hat{c}^i_{f,t+1}}\frac{1}{g_{b,t}}\right)^{\rho}(1+\pi\lambda_{f,t+1})m_{f,t+1}g_{b,t}\right], \end{aligned} \tag{40}$$

$$r_{f,t} = \alpha \frac{\hat{Y}_{f,t}}{u_{f,t}}, \tag{41}$$

$$r_{b,t} = \alpha \frac{\hat{Y}_{b,t}}{u_{b,t}}, \tag{42}$$

$$\hat{w}_{f,t} = (1-\alpha) \frac{\hat{Y}_{f,t}}{(1-\pi) l_{f,t}}, \tag{43}$$

$$\hat{w}_{b,t} = (1-\alpha) \frac{\hat{Y}_{b,t}}{(1-\pi) l_{b,t}}, \tag{44}$$

$$\hat{Y}_{f,t} = \pi \hat{c}^i_{f,t} + (1-\pi) \hat{c}^s_{f,t} + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1 - \delta(u_{f,t})) + m_{f,t}}{1 - \phi q_{f,t}}, \tag{45}$$

$$\hat{Y}_{b,t} = \pi \hat{c}^i_{b,t} + (1-\pi) \hat{c}^s_{b,t} + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1 - \delta(u_{b,t})) + m_{b,t}}{1 - \phi q_{b,t}}, \tag{46}$$

$$g_{f,t} = 1 - \delta(u_{f,t}) + \pi \frac{u_{f,t} r_{f,t} + \phi q_{f,t} (1 - \delta(u_{f,t})) + m_{f,t}}{1 - \phi q_{f,t}}, \tag{47}$$

$$g_{b,t} = 1 - \delta(u_{b,t}) + \pi \frac{u_{b,t} r_{b,t} + \phi q_{b,t} (1 - \delta(u_{b,t})) + m_{b,t}}{1 - \phi q_{b,t}}, \tag{48}$$

$$\lambda_{f,t} = \frac{q_{f,t} - 1}{1 - \phi q_{f,t}}, \tag{49}$$

and

$$\lambda_{b,t} = \frac{q_{b,t} - 1}{1 - \phi q_{b,t}} \tag{50}$$

where subscripts f and b denote realizations of the variables in a fundamental and bubble regime, respectively; for instance, $\hat{Y}_{f,t}$ is the realization of $\hat{Y}_t$ in a fundamental regime.

The impulse response functions are calculated by linearizing the equations (29) to (50) around $\hat{Y}_{f,t} = \hat{Y}_f$, $\hat{c}^i_{f,t} = \hat{c}^i_f$, $\hat{c}^s_{f,t} = \hat{c}^s_f$, $l_{f,t} = l_f$, $g_{f,t} = g_f$, $q_{f,t} = q_f$, $\lambda_{f,t} = \lambda_f$, $u_{f,t} = u_f$, $r_{f,t} = r_f$, $\hat{w}_{f,t} = \hat{w}_f$, $\hat{Y}_{b,t} = \hat{Y}_b$, $\hat{c}^i_{b,t} = \hat{c}^i_b$, $\hat{c}^s_{b,t} = \hat{c}^s_b$, $l_{b,t} = l_b$, $g_{b,t} = g_b$, $q_{b,t} = q_b$, $\lambda_{b,t} = \lambda_b$, $u_{b,t} = u_b$, $r_{b,t} = r_b$, $\hat{w}_{b,t} = \hat{w}_b$ and $m_{b,t} = m_b$.

8.3 Existence Condition

From the discussion above, it should be apparent that depending on the degree of financial tightness, bubbles may or may not be valuable. In this section, we highlight other elements that may affect bubbles' valuation.

8.3.1 Permanent Bubble

The steady-state investment condition (23) is useful to understand when bubbles arise (are valued positively). To this end, let's re-write it as follows:

$$m = \hat{\imath}(1 - \phi q) - ur - \phi q(1 - \delta(u)). \quad (51)$$

Here, m and $\hat{\imath}$ are the size of the bubbles and investment relative to the capital stock, i.e., $m_t = \tilde{p}_t M/K_t$ and $\hat{\imath}_t = i_t/K_t$, in the steady state respectively. The first term in the right-hand side of equation (51) is the down payment each investor pays for investment. The second term is the rental rate of capital, and the third term is the proceeds from selling capital up to the limit. Therefore, this equation says that bubbles have positive valuation (the left-hand side is positive) if and only if the amount of liquidity an investor can withdraw from capital is less than the amount of liquidity investors need to undertake an investment project.

To convey more intuition, let's assume that utilization is 1 and there is full depreciation. Under these assumptions, equation (51) is rewritten as

$$m = g(1 - \phi q) - r \quad (52)$$

because $\hat{\imath} = g$ where g is the growth rate of the economy in the steady state. Bubbles are valued when the rental rate of capital is sufficiently low. This implication is in line with the previous work on bubbles; if we further assume that ϕ is equal to $\phi = 0$, the first term in the right-hand side collapses to g, and $g > r$ is the familiar dynamic inefficiency condition for the existence of bubbles in OLG models.

If ϕ is strictly positive, investors can borrow money from savers using capital as collateral. By making the first term in the right-hand side smaller, a larger value of ϕ makes it more difficult to support bubbles. This implication is also in line with previous work; e.g., Tirole (1982) shows that bubbles cannot arise in infinite horizon economies in which agents can borrow and lend freely. In other words, a tight enough friction in the financial market is necessary for the economy to have a bubble equilibrium.

8.3.2 Recurrent Bubble

Let us briefly discuss the existence condition when bubbles come and go. Assuming full depreciation and fixing the utilization at one, we arrive at the following expression,

$$m_b = \hat{\imath}_b(1 - \phi q_b) - r_b.$$

Other things being equal, bubbles are sustained (m_b is positive) when the liquidity constraint is tight, the rental price of capital is high, and/or the investment (and hence the growth rate) in the bubble regime is high. These implications are similar to those in the permanent bubble model.

But people take into account the possibility of the bubble bursting when they are in the bubble regime, evaluating assets accordingly. The opposite is true in the fundamental regime. Therefore, both prices and behaviors are affected not only by the actual occurrence of the regime switch but also by the sheer possibility of the regime switch. For instance, under full depreciation the steady-state price of equity in the bubbly regime is

$$q_b = (1 - \sigma_b)\, \beta g_b^{-\rho} \left(r_b + \pi \lambda_b r_b\right) + \sigma_b \beta \left(\frac{\hat{c}_b^i}{\hat{c}_f^i} \frac{1}{g_b} \right)^{\rho} \left(r_f + \pi \lambda_f r_f\right).$$

Clearly, the dynamic link between the two regimes makes the existence condition more complicated, but it sheds a new light on the study of bubbles.

8.4 Impulse Responses

Table 3 shows responses to a 1 percentage point change in either productivity or preference shocks. Their autocorrelations are assumed to be 0.77 and 0.3, respectively, roughly equal to our benchmark estimates. We report contemporaneous responses on impact of the shock alone because they are enough to summarize the impulse responses; remember that there are no endogenous state variables in our model once endogenous variables with trend are divided by K_t, implying that both the regime $z_t \in \{f, b\}$ and the levels of the exogenous shocks $\{a_t, b_t\}$ are sufficient to pin down detrended-endogenous variables.

A productivity shock temporarily raises GDP growth but is nearly neutral for the credit-to-GDP ratio. Credit expands with GDP because the productivity shock raises asset prices, which allows investors to obtain funds easily. Not surprisingly, the productivity shock temporarily accelerates capital accumulation too.

In contrast, a preference shock temporarily raises the credit-to-GDP ratio but is nearly neutral for GDP growth. The shock decreases the subjective discount factor, but because it is mean reverting, it decreases the discount factor in the near future disproportionately. The shock therefore effectively makes households more patient; after the shock, households put relatively larger weights to the utility flows in the distant future compared those in the near future. This is why the households increase investment and decrease consumption after the preference shock. Asset prices also increase for the same reason, resulting in a temporary credit expansion. The preference shock temporarily accelerates capital accumulation too.

Comparing responses across regimes, we see that responses are larger in the bubbly regime than in the fundamental regime. Bubbles amplify the impact of the shocks because the bubble size positively responds to the shocks, supplying more funds to investors. Nonetheless, responses reported in the table are generally similar across regimes. Unlike the regime-dependence in the steady states, the regime-dependence in the short-run fluctuations is weak.

	Bubbly Regime		Fundamental Regime	
	Productivity	Preference	Productivity	Preference
GDP growth	1.13%	-0.01%	1.10%	-0.05%
credit-to-GDP	0%	0.71%	-0.14%	0.74%
output-to-capital	1.13%	-0.01%	1.10%	-0.05%
consumption-to-capital	1.03%	-0.29%	1.03%	-0.27%
investment-to-capital	1.42%	0.84%	1.41%	0.88%
hours	0.07%	0.21%	0.06%	0.17%
utilization	0.24%	-0.44%	0.19%	-0.48%
capital price	0.89%	0.64%	0.97%	0.68%
bubble size-to-capital	1.36%	0.74%	0%	0%
capital growth	0.05%	0.06%	0.04%	0.04%

Table 3: Effects of Productivity and Preference Shocks. Responses to a 1 percentage point change in either productivity or preference shocks are displayed. Their autocorrelations are 0.77 and 0.3 respectively.

8.5 Data

In this section, we explain the observables used to estimate the model. The data consist of quarterly GDP growth and the credit-to-GDP ratio for the period 1960.Q1 - 2017.Q4. In the main text, we use the post-1984 data because of a technical complication related to growth in the pre-Great Moderation period. The next section discusses this issue in detail and demonstrates that the main results in the paper are robust to the longer sample. The data come from the St. Louis Fed's FRED database. For credit, we use the series' total credit to the private non-financial sector. The sample is rich in that contains events such as the strong growth in the 1960s; the three greats: Inflation, Moderation, and Recession; as well as the housing boom and subsequent financial crisis. We use the raw unfiltered series for GDP. We pre-filtered the credit-to-GDP series because of a secular trend in the data that is not present in our model (blue line in Figure 11).[16] We tried different trends from linear, quadratic, and polynomial. Based on the adjusted R^2, we settled on a linear trend (filtered data in red in Figure 11; see the main text for a discussion of the properties of the filtered series). This is a reasonable step because we are interested in understanding how the fluctuations around this trend are influenced by the presence of bubbles and the other regimes in the model. Furthermore, this de-trending approach is a standard approach in policy institutions such as the Federal Reserve System, when analyzing the evolution of credit in the economy (Bassett, Daigle, Edge, and Kara (2015)). An alternative is to introduce a trend for the series. But because we lack a theory behind the trend, we choose not to include it.

[16] Interestingly, this trend is present in different subcategories of credit such as home mortgage, consumer credit, and commercial mortgages.

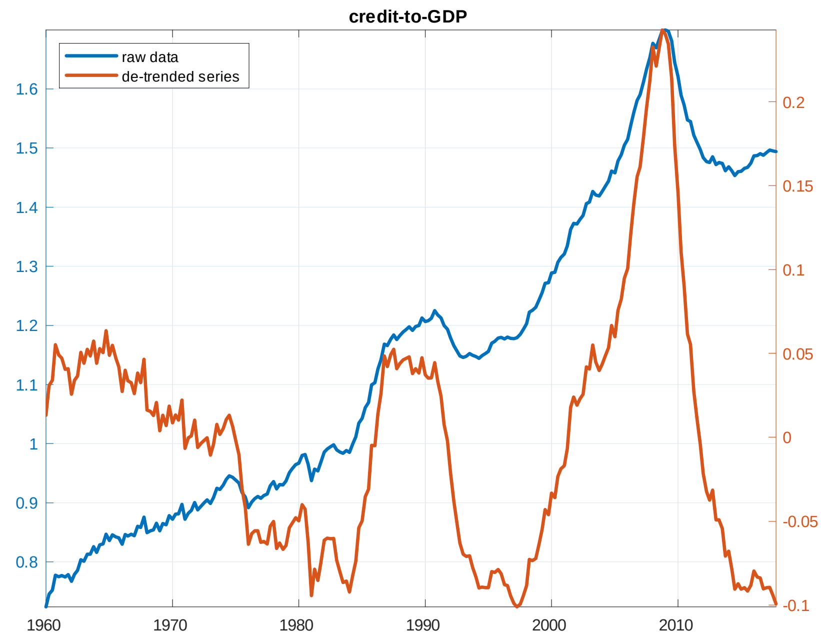

Figure 11: Credit-to-GDP in the U.S.

8.6 Alternative Identification Strategies

As explained in the paper, for the liquidity constraint of $\phi = 0.19$, the average growth rates and credit-to-GDP are off the values in the data seen during the bubbly episode in the 2000s. One possibility, used in the paper, is to introduce a constant and estimate it to offset the difference. Alternatively, one can "arbitrarily" reduce the liquidity constraint to match the average growth rate during the credit bubble. Under this specification, we estimate only the persistence and volatility of the structural shocks. Figure 12 shows the estimated path of the probability of the fundamental (upper panel) and bubbly (lower panel) regimes. Clearly, the paths are consistent with those reported in the paper.

In the main text, we estimate the regimes using the sample 1984.Q1 - 2017.Q4. One can extend the sample to include the pre-Great Moderation era 1960.Q1 - 1983.Q4 but this brings a complication. Growth was strong during that period and credit-to-GDP was above average. Through the lens of our benchmark model, this points to a bubble. However, most economic observers would agree that there was no credit-driven bubble during those years (Figure 11). To cope with this issue, we add a third regime that allows for high growth and average credit. Figure 13 shows the probabilities of each regime from this alternative model. As one can see, the main message remains. The high growth/high credit of the 2000s was most likely associated with the occurrence of a bubble in the economy. We also see that the economy spent most of the 1960s and 1970s in the third regime.

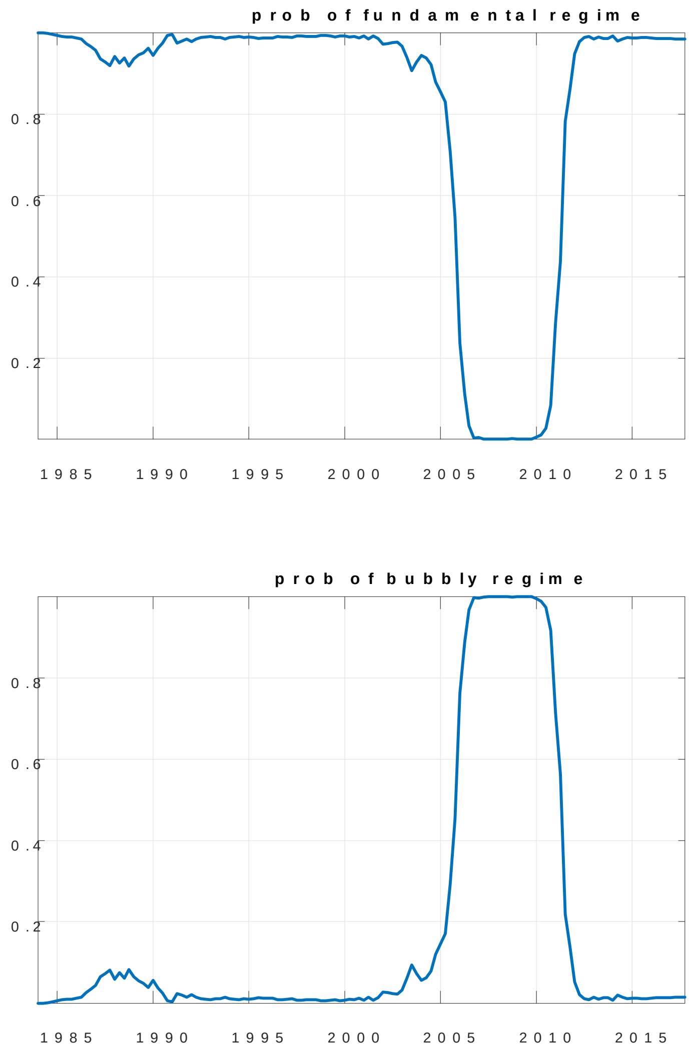

Figure 12: Regime Probabilities with Tighter Liquidity

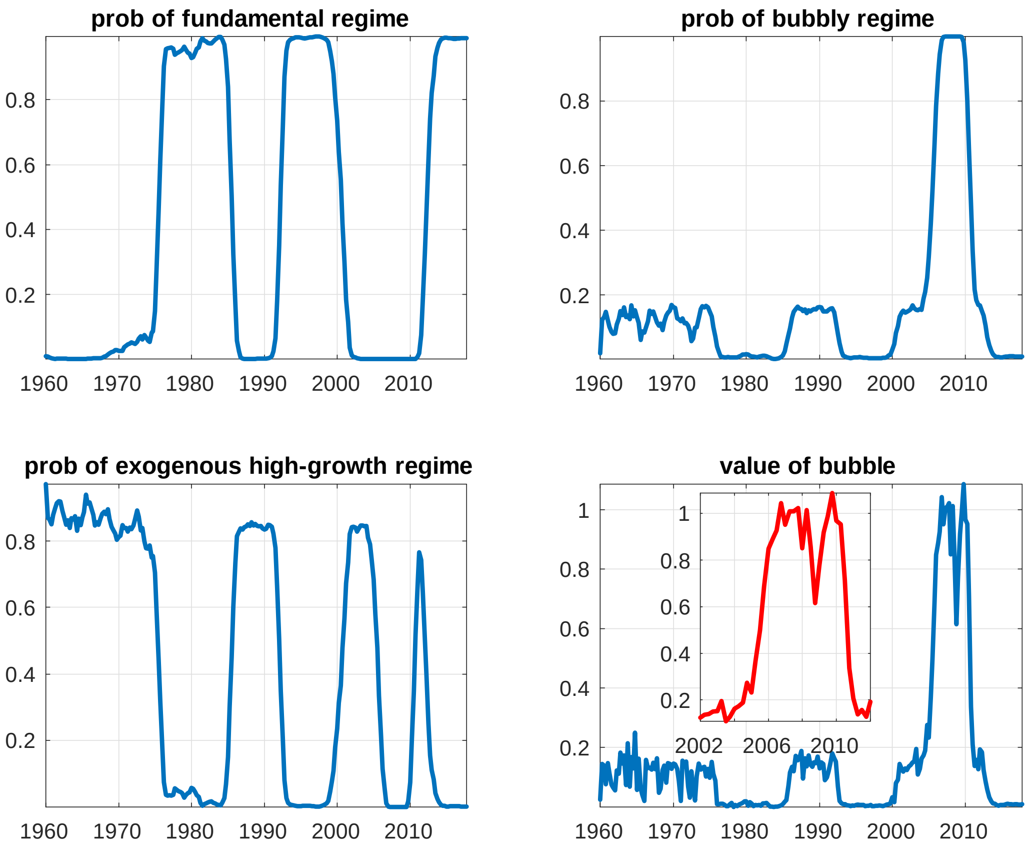

Figure 13: Regime Probabilities Extended Sample