Network Externalities and Market Dominance*

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Abstract

We develop a framework to study optimal pricing and price competition in the presence of multiple equilibria caused by network externalities. This framework provides a simple approach to equilibrium selection, based upon consumers' "impulses," that yields concise predictions regarding when firms will be "in" and when "out." We highlight the role of past consumption and "influencers" in shaping impulses, and provide a unified explanation for a variety of stylized facts including why: (i) it is difficult to become "in"; (ii) the "in" position is fragile; (iii) "in" firms are not asleep in the sense that they continuously raise quality and keep prices low; and (iv) "in" firms acquire startups to entrench their position.

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1 Introduction

A key goal in economics is to understand the nature of competition. Market efficiency, consumer welfare, the intensity of innovation, and appropriate anti-trust policy all depend upon how competition plays out.[1] Competition differs markedly between the "old economy" (where average costs are U-shaped and demand is downward-sloping) and the "new economy" (where average costs are decreasing and demand is subject to network externalities). The new economy is of increasing importance — it includes, for instance, the five largest publicly traded companies (Apple, Google/Alphabet, Microsoft, Amazon, Facebook) — yet it is not fully understood.

Our goal is to offer a simple framework to model pricing and competition in this new economy. The challenge is how to handle network externalities in demand, as they give rise to multiple equilibria. In our model, consumer "impulses" determine whether sales are high or low. These impulses depend upon past consumption and the behavior of "influencers" and are, in principle, observable.

We are particularly interested in understanding three features of the new economy. First, winning firms serve a disproportionate share of their markets, with a large size gap between them and their closest rivals. For instance, Google's current share of web, mobile, and in-app searches is 90.8% while Amazon's share of the US e-book market is 83%.[2,3]

Second, it is difficult to become a winner and yet success is so fragile that it can vanish overnight. The difficulties of becoming popular (going from being "out" to being "in") are illustrated by Microsoft's search engine Bing, which despite years of sizeable investments remains much smaller than the current superstar Google.

[1] For instance, there is a long tradition in economics of analyzing the relationship between market structure and innovation, dating to the celebrated contributions of Schumpeter (1942) and Arrow (1962) Arrow (1962).

[2] See Jeff Desjardin, "How Google retains more than 90% of market share," Business Insider, April 24, 2018, retrieved from `http://www.businessinsider.com`, and Mark Gurman, "Apple's Getting Back Into the E-Books Fight Against Amazon," Bloomberg, January 25, 2018, retrieved from `http://www.bloomberg.com`.

[3] In 2002, Bill Gates summarized his ambitions as follows: "We look for opportunities with network externalities — where there are advantages to the vast majority of consumers to share a common standard. We look for businesses where we can garner large market shares, not just 30-35%." (Rivkin and Van den Steen (2009))

The ease with which a successful firm can suddenly fail (go from "in" to "out") is illustrated by the web browser Netscape, which despite its initial dominant status was overtaken quite suddenly by Microsoft's Internet Explorer.[4]

Finally, despite their seemingly-dominant position, winners are not asleep. They tend to continuously raise quality (for example, by purchasing new startups) while at the same time keeping their prices low, sometimes even below average cost. For example, despite serving around 70% of the US ride-sharing market, Uber invests heavily in its mapping technology and, at the same time, has kept its prices so low that it has been unable to recoup its costs.[5] Similarly, Google invests heavily in its search algorithm to fend off wealthy, though much smaller, rivals like Microsoft Bing.

As we shall see, our model readily accommodates these observations. In addition, it provides a new perspective on firms' incentives to innovate and acquire startups, the extent of market power of large firms, the time-path of prices, and the role of influencers.

There is a large literature on network externalities. The classic models of Katz and Shapiro (1985) and Becker (1991) show that these externalities can generate demand curves with both downward- and upward-sloping regions, depending on whether the traditional price effect or the network effect dominates. Such demand curves admit the possibility of multiple equilibria.

Our approach to equilibrium selection relies upon level-k thinking (e.g. Stahl and Wilson (1994), Nagel (1995), Ho et al. (1998), Crawford (2003), Crawford and Iriberri (2007)) and, in particular, on the notion of "introspective equilibrium" developed by Kets and Sandroni (2017). The idea is that players' impulses (or, level-0 behavior) affect the ultimate equilibrium selection. Our innovation is to apply this framework to the new economy environment and to take a stance on how impulses form and change.

We also build upon the growing literature on equilibrium selection in markets with network externalities. Caillaud and Jullien (2003) introduce the idea of selection

[4]See Rivkin and Van den Steen (2009).

[5]See Deirdre Bosa, "Lyft claims it now has more than one-third of the US ride-sharing market," CNBC, May 14, 2018, retrieved from `http://www.cnbc.com`, and Len Sherman, "Why Can't Uber Make Money?," Forbes, December 14, 2017, retrieved from `http://www.forbes.com`.

based upon "favorable beliefs" (which is a notion of focality); they assume that the incumbent firm/platform is focal. Halaburda et al. (2018) consider a dynamic model of platform competition in which a firm's current focality — modeled as being all or nothing — depends on past sales. They derive conditions under which initial focality may allow an inefficient firm to survive. Halaburda and Yehezkel (2018) extend this framework to the case where focality can take intermediate values and thus firms can enjoy a partial belief advantage. In both Halaburda et al. (2018) and Halaburda and Yehezkel (2018), consumers are homogeneous. Argenziano and Gilboa (2012) consider dynamics in an abstract model without firms or pricing decisions. Biglaiser and Cremer (2016) consider dynamic competition between two platforms in an environment where migration between platforms is affected by coordination failures.[6]

As in this literature, we let focality evolve over time. What distinguishes our model is that it accommodates heterogeneous consumers; and our concept of focality is endogenous and micro-founded on level-k thinking. Our focality criterion also allows influencers, as well as past success, to impact focality.[7]

Also related is the literature on "switching costs," beginning with Klemperer (1987) (see also Klemperer (1995), Fudenberg and Tirole (2000), and Farrell and Klemperer (2007)). Switching costs generate incumbency advantage and mean that transitioning from "out" to "in" may require charging a low initial price. In our model, there is an incumbency advantage despite the absence of switching costs. Moreover, in contrast to switching-cost models, transitions can occur very quickly, may involve quantity overshooting, and are triggered by influencers as well as prices.

A broader literature on platforms, initiated by Rochet and Tirole (2003), analyzes markets that are multi-sided and involve externalities within and between sides (e.g. Rochet and Tirole (2006), Armstrong (2006), and Weyl (2010)). We connect our theory to this literature in Section 5.

Finally, there is a growing applied literature on the new economy. For instance,

[6] Ambrus and Argenziano (2009) use coalitional rationalizability to select equilibria. They restrict the amount of coordination failure across consumers and show that multiple asymmetric networks can exist in equilibrium.

[7] In the special case where consumers are homogeneous and there are no influencers, our equilibrium selection coincides with that of Halaburda et al. (2018).

Gompers and Lerner (1999) show that a sizeable share of R&D investment is done by new-economy startups. Gans et al. (2002) study incumbents' expropriation of the intellectual capital of startups, and the resulting impact on incentives to innovate. There is also an emerging debate in law and economics on appropriate anti-trust policy in the new economy. The so-called "New Brandeisian Movement" (see Khan (2017)) argues that there is too much focus on short-run consumer welfare, which misses the possibility that a firm may raise prices after building up a network (for the classic welfarist approach, see Kovacic and Shapiro (2000)). Finally, Brynjolfsson and McAfee (2014) argue that both network externalities and income inequality are magnified by artificial intelligence.[8] One of our goals is to inform this type of work.

The remainder of the paper is organized as follows. Section 2 presents our baseline model with a single firm. Section 3 considers price competition between two firms. Section 4 considers the special case of a piecewise linear demand curve. Section 5 extends our model to multi-sided markets, and Section 6 concludes.

2 Monopoly Case

To begin, consider a setting with a single firm (a monopolist). We assume that each period $t = 1, 2, ..., T$ the firm chooses a price p^t. The marginal cost of production is constant, which we normalize to zero.

There is a set of heterogeneous consumers. Consumer i's demand at time t, denoted q_i^t, depends upon the price p^t and upon aggregate consumption Q^t; that is, $q_i^t = D_i(p^t, Q^t)$. We assume D_i is decreasing in p^t and increasing in Q^t, reflecting the presence of positive network externalities.

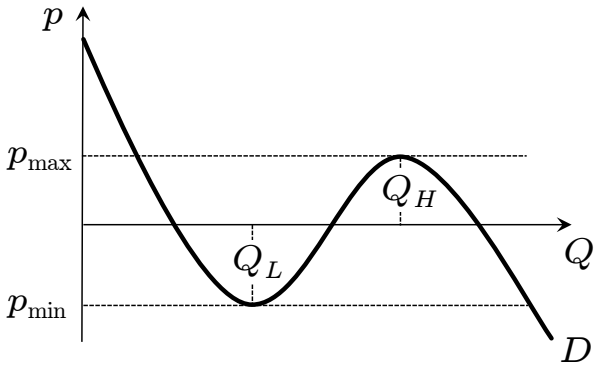
For any given p^t, an equilibrium quantity Q^{t*} satisfies:

$$Q^{t*} = \sum_i D_i \left(p^t, Q^{t*} \right).$$

In the spirit of Katz and Shapiro (1985) and Becker (1991), we will focus attention

[8] See Edelman (2015, June 21, 2017) and Edelman and Geradin (2016) for examples of how nefarious or illegal practices can be used to harness network externalities.

Figure 1: In/Out Demand Curve



on cases where this relationship gives rise to demand curves with regions of positive slope. We are interested, specifically, in demand curves of the type shown in Figure 1, which we term "in/out." The idea, loosely speaking, is that when total consumption is low, the network effect is weak and demand has a standard shape (i.e. a negative slope). When total consumption exceeds a first critical level (Q_L in the figure), the network externality begins to dominate, causing marginal value to grow with quantity. When total consumption exceeds a second critical level (Q_H in the figure), the network externality is exhausted and demand again has a standard shape. The next section formalizes this possibility.

2.1 Micro-foundation for In/Out Demand

Assume there is a continuum of consumers ($i \in [0,1]$). Consumer i has an idiosyncratic taste θ_i for the monopolist's good that is distributed iid according to a distribution with cdf F.

Consumers face a binary choice whether to consume. We normalize to zero the utility from not consuming and assume the utility from consuming is $\theta_i + \mu + \alpha \cdot Q - p$, where μ is the quality of the monopolist's good relative to the outside option, Q is the population mass currently consuming, and p is the price. The parameter α captures the size of the network externalities.

Under these assumptions, agent i consumes if and only if $\theta_i + \mu + \alpha \cdot Q - p \geq 0$. Therefore, the agents who consume are those whose idiosyncratic taste for the good

exceeds a threshold $\hat{\theta}$, where:

$$\hat{\theta} = \underbrace{p - \mu}_{\text{effective price}} - \underbrace{\alpha \cdot Q}_{\text{network externality}}. \tag{1}$$

This threshold is increasing in the good's "effective price" (the price net of quality) and decreasing in the size of the network externality.

Observe that aggregate demand Q is equal to the mass of consumers above the threshold:

$$Q = 1 - F(\hat{\theta}). \tag{2}$$

Combining equations (1) and (2), we find that:

$$Q = 1 - F(p - \mu - \alpha \cdot Q). \tag{3}$$

Rearranging terms, we obtain (inverse) demand:

$$p^d(Q) = F^{-1}(1 - Q) + \alpha \cdot Q + \mu. \tag{4}$$

Notice from equation (4) that μ, the good's quality relative to the outside option, shifts the demand curve vertically.

We can differentiate equation (4) to obtain an expression for the slope of demand:

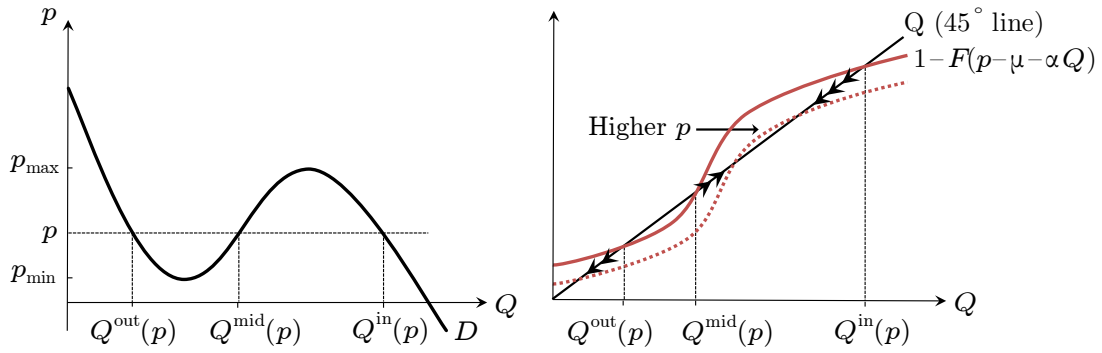
$$\frac{dp^d(Q)}{dQ} = \underbrace{\alpha}_{\text{network externality}} - \underbrace{\frac{1}{f(F^{-1}(1-Q))}}_{\text{distribution of consumers' tastes}}. \tag{5}$$

The slope depends both upon the size of network externalities (term 1) and the distribution of consumers' tastes (term 2).

From here, it is easy to obtain an in/out demand curve. Suppose, for instance, F is normal with mean 0 and variance σ^2. The demand curve has a maximum slope of $\alpha - \frac{1}{f(0)} = \alpha - \sqrt{2\pi\sigma^2}$ (at $Q = \frac{1}{2}$) and a minimum slope of $\alpha - \frac{1}{f(\pm\infty)} = -\infty$ (at $Q = 0$ and $Q = 1$). Therefore, demand has an in/out shape, as in Figure 1, if and only if the network externalities are sufficiently large ($\alpha > \sqrt{2\pi\sigma^2}$).[9]

[9]If consumers' tastes instead follow a uniform distribution, the demand curve has a constant

Figure 2



(a) In/Out Demand Curve. **(b)** Demand solves: $Q = 1 - F(p - \mu - \alpha Q)$.

More generally, if F has support $\mathbb{R}$ and the pdf is single-peaked, there exists a threshold $\hat{\alpha} > 0$ such that:

1. Demand is downward-sloping if $\alpha \leq \hat{\alpha}$.
2. Demand is in/out if $\alpha > \hat{\alpha}$.[10]

To understand the in/out shape, consider Figures 2a and 2b. Figure 2a depicts an in/out curve and Figure 2b depicts the solution to equation (3), which gives rise to this curve. Observe that, in Figure 2a, if the firm sets an intermediate price ($p_{\min} < p < p_{\max}$), there are three intersections with the in/out curve — hence three Nash equilibria. We will denote these equilibria as $Q^{out}(p)$, $Q^{mid}(p)$, and $Q^{in}(p)$ (where $Q^{out}(p) < Q^{mid}(p) < Q^{in}(p)$).[11]

Figure 2b formalizes why, at intermediate prices, there are three possible quantities demanded and shows the impact of a change in price. $Q^{out}(p)$ and $Q^{in}(p)$ both decrease when p rises; correspondingly, the demand curve is downward sloping at $Q^{out}(p)$ and $Q^{in}(p)$. In contrast, $Q^{mid}(p)$ increases when p rises; correspondingly, the demand curve is upward sloping at $Q^{mid}(p)$.

slope because $f(\theta)$ is constant.

[10]Katz and Shapiro (1985) focus on demand curves with an upside-down U-shape. Their micro-foundation is slightly different from ours: they assume idiosyncratic tastes are uniformly distributed (F uniform) and obtain an upward-sloping region by assuming that the network externalities are nonlinear (i.e., α is a function of Q). Becker (1991) focuses both on demand curves with an upside-down U-shape and on in/out demand curves; he does not offer a micro-foundation.

[11]Notice that $Q^{mid}(p) = Q^{out}(p)$ when $p = p_{\min}$, and $Q^{mid}(p) = Q^{in}(p)$ when $p = p_{\max}$.

2.2 Equilibrium Selection

In this section, we offer a simple theory of equilibrium selection. This theory will be central to our analysis.

We begin by invoking a general refinement of Nash equilibrium due to Kets and Sandroni (2017) called "introspective equilibrium." Introspective equilibrium is based upon level-k thinking (see Crawford et al. (2013) for a survey). Agents have exogenously-given "impulses," which determine how they react at level 0. At level $k > 0$, each agent formulates a best response to the belief that opponents are at level $k-1$. Introspective equilibrium is defined as the limit of this process at $k \to \infty$. This approach to equilibrium selection nests a wide range of refinement concepts, corresponding to different assumptions about agents' impulses.[12]

We apply introspective equilibrium to our setting as follows.

Definition 1 (Introspective Equilibrium for In/Out Demand)**.**

Fix a time period t. Players are endowed with level-0 choices $\overline{q}_{i0}$ called (individual) impulses, which lead to an (aggregate) impulse $\overline{Q}_0$. For any given p, an introspective equilibrium (q_i^*, Q^*) is constructed as follows:

1. *Level $k = 1, 2, ...,$ denoted $(\overline{q}_{ik}, \overline{Q}_k)$, is obtained by letting each consumer best-respond to price p and the belief that other consumers are at level $k-1$.*

2. *An introspective equilibrium is the limit as $k \to \infty$:*

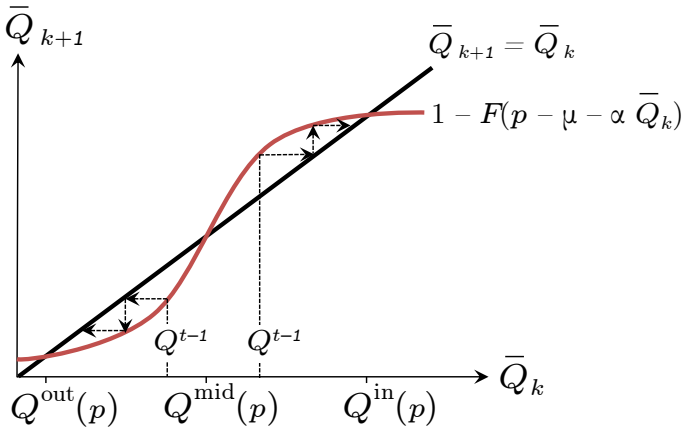
$$(q_i^*, Q^*) = \lim_{k \to \infty} (\overline{q}_{ik}, \overline{Q}_k).$$

The impulses $(\overline{q}_{i0}, \overline{Q}_0)$ play a key role in our model. To begin, we assume that each agent's impulse in period t is to do what she did in the previous period; that is $\overline{q}_{i0} = q_i^{t-1}$ and $\overline{Q}_0 = Q^{t-1}$. We take agents' impulses in the first period as exogenous and denote them as q_i^0 and Q^0.

Lemma 1 derives the introspective equilibrium as a function of the aggregate impulse (Q^{t-1}).

[12] Risk dominance, for instance, corresponds to the case where agents are uncertain about each others' impulses.

Figure 3: $\overline{Q}_{k+1} = 1 - F(p - \mu - \alpha\overline{Q}_k)$



Lemma 1. *Fix a time t and suppose the aggregate impulse is Q^{t-1}. When $p_{\min} \leq p \leq p_{\max}$, the unique introspective equilibrium is:*

$$Q^*\left(p, Q^{t-1}\right) = \begin{cases} Q^{in}(p), & \text{if } Q^{t-1} > Q^{mid}(p). \\ Q^{mid}(p), & \text{if } Q^{t-1} = Q^{mid}(p). \\ Q^{out}(p), & \text{if } Q^{t-1} < Q^{mid}(p). \end{cases}$$

When $p > p_{\max}$ or $p < p_{\min}$, $Q^*\left(p, Q^{t-1}\right)$ is the unique solution to equation (3).

Proof of Lemma 1. The proof is easy to derive. Suppose first that $p_{\min} \leq p \leq p_{\max}$. From equation (3), we obtain the evolution of aggregate consumption between levels k and $k+1$:

$$\overline{Q}_{k+1} = 1 - F(p - \mu - \alpha\overline{Q}_k). \tag{6}$$

Figure 3 corresponds to equation (6). It shows how, starting from an initial impulse (Q^{t-1}), the aggregate consumption level evolves.

Observe that, if there is a high initial impulse to consume ($Q^{t-1} > Q^{mid}(p)$), aggregate consumption increases between levels 0 and 1. Intuitively, the high level of consumption at level 0 drives more agents to consume at level 1. Consumption continues to increase between levels 2 and 3, 3 and 4, and so forth, reaching $Q^{in}(p)$ in

the limit. Hence, when $Q^{t-1} > Q^{mid}(p)$, the introspective equilibrium is $Q^{in}(p)$. By a similar logic, aggregate consumption falls between successive levels when $Q^{t-1} < Q^{mid}(p)$. When $Q^{t-1} < Q^{mid}(p)$, the introspective equilibrium is $Q^{out}(p)$.

Finally, when $p > p_{\max}$ or $p < p_{\min}$, the proof is very similar. The only difference is that there is a single intersection in the analog of Figure 3. Q.E.D.

Proposition 1 is the main result in this section. This result, which is a direct consequence of Lemma 1, says that the firm faces one of three negatively-sloped demand curves (shown in Figure 4).

Proposition 1. *In any period, the firm faces one of three downward-sloping demand curves (depending upon Q^{t-1}):*

1. *"In" Demand Curve ($Q^{t-1} \geq Q_H$): Figure 4a.*
2. *"Out" Demand Curve ($Q^{t-1} \leq Q_L$): Figure 4b.*
3. *"Between" Demand Curve ($Q_L < Q^{t-1} < Q_H$): Figure 4c.*

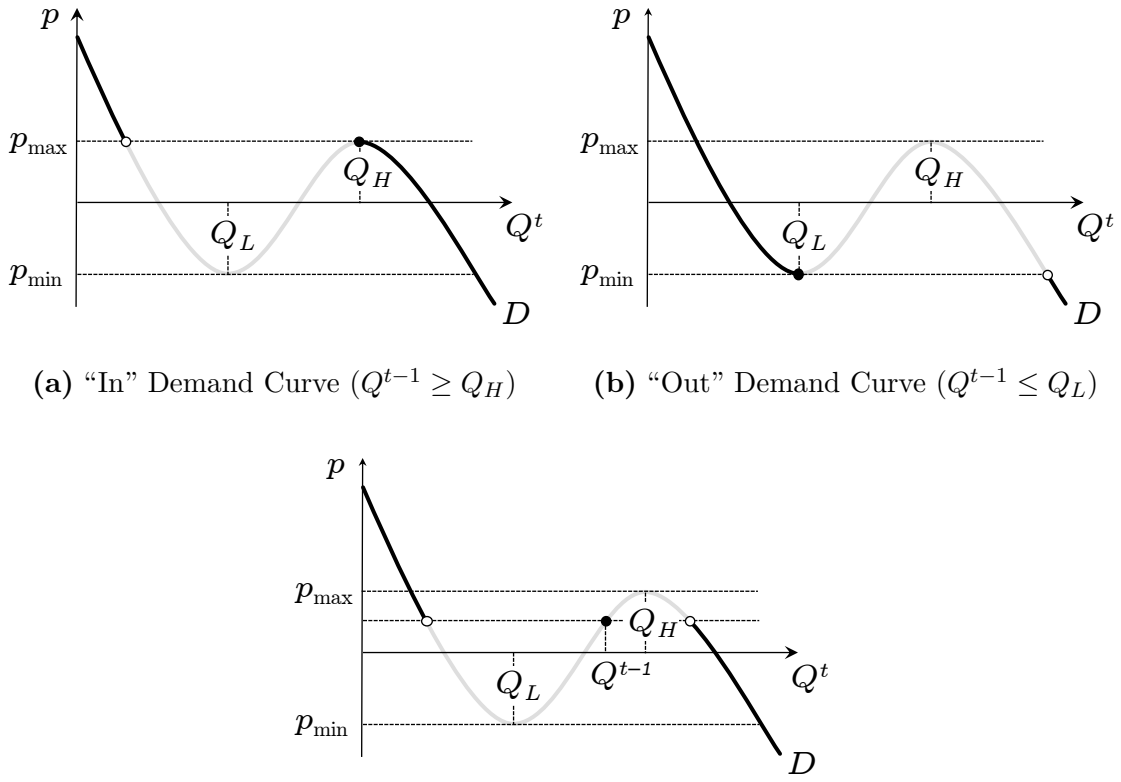
(All omitted proofs are in the Appendix.)

If $Q^{t-1} \geq Q_H$, the monopolist faces the best possible equilibrium selection (that is, the "in" demand curve). In this case, we will say that the firm is "in." If $Q^{t-1} \leq Q_L$, the monopolist faces the worst possible equilibrium selection (that is, the "out" demand curve). In this case, we will say that the firm is "out." And if $Q_L < Q^{t-1} < Q_H$, the monopolist faces an intermediate equilibrium selection (that is, the "between" demand curve). In this case, we will say that the firm is "between."

If, in period t, the firm sells no less than Q_H, we will say the firm *ends* the period "in." Similarly, if the firm sells no more than Q_L, we will say that the firm ends the period "out." And, if the firm sells an intermediate amount ($Q_L < Q^t < Q_H$), we will say that the firm ends the period "between."

To illustrate the process of transitioning from "out" to "in," suppose the firm begins period t "out" ($Q^{t-1} \leq Q_L$). If the firm sets a price above $p_{\min}$, it will sell less than Q_L and consequently continue to face an "out" demand curve in period $t+1$. If instead the firm sets a price below $p_{\min}$, it will sell more than Q_H and consequently

Figure 4



(a) "In" Demand Curve ($Q^{t-1} \geq Q_H$) **(b)** "Out" Demand Curve ($Q^{t-1} \leq Q_L$)

(c) "Between" Demand Curve ($Q_L < Q^{t-1} < Q_H$)

face an "in" demand curve in period $t+1$. This allows the firm to raise its price as far as $p_{\max}$ in period $t+1$ without losing its "in" status.

2.3 Optimal Pricing

We are now in a position to formally state the monopolist's problem and describe optimal pricing.

Profits in period $t = 1, 2, ..., T$ are $\pi^t = p^t \cdot Q^t$, where $Q^t = Q^*(p^t, Q^{t-1})$ (and $Q^*(p^t, Q^{t-1})$ is defined as in Lemma 1). We take Q^0, the initial consumption level or impulse, as given. In each period, the monopolist chooses price to maximize the discounted sum of future profits, using discount factor δ.

Proposition 2. *Suppose the firm is myopic ($\delta = 0$). Depending upon the shape of*

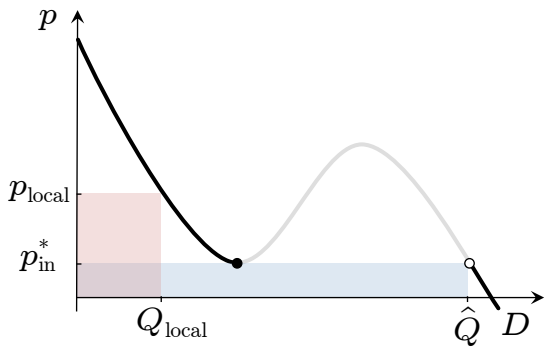
the demand curve (which is a function of α, μ, and F), there are three cases:

(i) It is optimal to go "in" in period t (choose p^t such that $Q^t \geq Q_H$) regardless of the impulse Q^{t-1}.

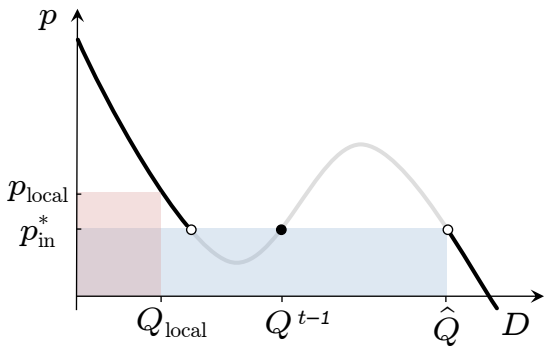
(ii) It is optimal to go "out" in period t (choose p^t such that $Q^t \leq Q_L$) regardless of the impulse Q^{t-1}.

(iii) It is optimal for "in" ("out") firms to stay "in" ("out"). "Between" firms go "in" ("out") if Q^{t-1} is above (below) a cutoff (Q^{myopic}).

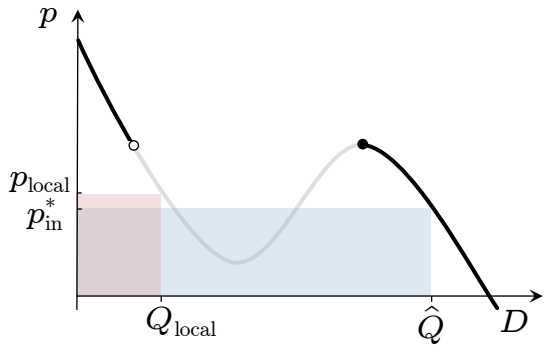
Figure 5: Optimal Pricing in the Myopic Case



(a) "Out" Demand ($Q^{t-1} \leq Q_L$)



(b) "Between" Demand ($Q_L < Q^{t-1} < Q_H$)



(c) "In" Demand ($Q^{t-1} \geq Q_H$)

Intuitively, because demand is discontinuous at the threshold price for going "out"/"in," the firm always chooses to end the period "out" or "in" (never "between"). Consequently, as illustrated in Figure 5, the firm will either end "out" and

choose price p_{local} (which is independent of Q^{t-1}) or end "in" and choose price p^*_{in} (which is weakly increasing in Q^{t-1}).[13]

Notice that the profits when ending "out" (denoted π_{local}) are independent of the impulse Q^{t-1} while the profits when ending "in" (denoted π^*_{in}) are weakly increasing in the impulse. Therefore, instance (i) of Proposition 2 arises when $\pi^*_{in}(Q^{t-1})$ is greater than π_{local} for all values of Q^{t-1}; instance (ii) arises when $\pi^*_{in}(Q^{t-1})$ is less than π_{local} for all values of Q^{t-1}; and instance (iii) arises when $\pi^*_{in}(Q^{t-1}) > \pi_{local}$ if and only if Q^{t-1} is above a cutoff (Q^{myopic}).

Proposition 3 compares the optimal pricing when $\delta > 0$ with the optimal pricing of a myopic firm.

Proposition 3. *Suppose the firm is not myopic. In instances (i) and (ii) in Proposition 2, the firm acts as if it were myopic. In instance (iii), the firm potentially acts non-myopically in the first period, but acts myopically after that. Specifically, if $Q^{non\text{-}myopic} < Q^0 < Q^{myopic}$, the firm charges a price below the myopic optimum in the first period so as to go "in" — a form of investment — and stays "in" in all subsequent periods. If instead $Q^0 > Q^{myopic}$ (respectively, $Q^0 < Q^{non\text{-}myopic}$), the firm behaves as if it were myopic and in every period goes "in" (respectively, "out").*

Proof of Proposition 3. In instance (i), even if the firm disregarded the future, it would choose the "in" price over the "out" price for any initial impulse. Concern about the future only makes the firm more inclined to choose the "in" price, so it does not change the firm's behavior.

In instance (ii), the firm does not value having a higher impulse tomorrow: since the "out" profits (π_{local}), which do not depend upon the impulse, always exceed the "in" profits. Consequently, the firm behaves as if it were myopic.

We now turn to instance (iii). Observe that a lower price today (weakly) benefits the firm tomorrow: since a lower price today results in a higher impulse-to-consume tomorrow. Consequently, concern about the future lowers the cutoff impulse for choosing the "in" price over the "out" price. Q.E.D.

[13] In panels (a) and (b) of Figure 5, p^*_{in} is depicted as being right at the threshold for the firm to go "in" but p^*_{in} may also be lower than that threshold. When p^*_{in} is right at the threshold, to circumvent an existence issue, we assume the firm sets $p^*_{in} = p^-_{\min}$, where $p^-_{\min}$ is a price just below $p_{\min}$.

A firm that chooses to go "in" may initially set a price below its long-term level. Once "in," the firm can raise its price and reap the benefits. Notice that quantity may initially overshoot the long-run level. This overshooting corresponds, for instance, to the opening night of a restaurant or play, where the goal is to fill the venue to capacity (quite possibly at a loss) to establish popularity.[14]

Remark: Price Dynamics

In our baseline model the price dynamics are very simple. Recall that, if the firm is initially "out," it may price below $p_{\min}$ for one period in order to go "in," and then price between $p_{\min}$ and $p_{\max}$ in every subsequent period.

More sophisticated dynamics are readily obtained by changing the definition of the impulse. Suppose, for example, that the period t impulse is a weighted average of past consumption levels: that is, it is given by $\lambda_1 Q^{t-1} + \lambda_2 Q^{t-2} + ...$ for some λ-weights. In this case, to transition from "out" to "in," the firm may need to price below the steady-state level for multiple periods. Moreover, during such periods, price need not be constant. One possibility, for instance, is that in the process of moving from "out" to "in," the firm spends several periods in the "between" position, while the impulse grows. During these transitional periods the "between" demand curve gradually improves, allowing the firm to charge a higher and higher price.

The strategy of pricing low initially to become "in" seems to be commonplace. For instance, according to Adam Cohen, to build up its network in its early days, PayPal "offered the service for free to both buyers and sellers and used millions of dollars in VC funds to hand out bounties of five dollars to everyone who signed up...Handing out free money was costly, but fairly effective: by the end of 1999, PayPal had signed up twelve thousand registered users."[15,16]

[14]The switching-cost literature gives one reason why firms would want to price low initially: doing so builds up a set of locked-in consumers. Our framework shows an additional reason: even in the absence of switching costs, pricing low initially builds up a product's perceived popularity, via the impulse. Observe also that quantity overshooting does not occur in switching-cost models.

[15]Cohen (2003), p. 228. PayPal raised its fees after building up its network. In 2017, the standard PayPal online transaction fee in the United States was 2.9% plus 30 cents. The "merchant rate" (available to those conducting $3,000 of business a month) was 1.9% plus 30 cents.

[16]This is by no means an isolated case. For example, in its "Free Basics Initiative," Facebook offers low-income consumers in developing countries free internet access to select websites.

2.4 Influencers

Suppose there is an agent, called an "influencer," who is connected to a fraction ϕ of the consumers and has the ability to set those consumers' impulses. Let $b \in \{0, 1\}$ denote whether the influencer sets the impulses of those consumers to "don't consume" or to "consume."

The influencer can play a pivotal role in determining whether the firm is "in" or "out," as the following corollary to Proposition 1 demonstrates.

Corollary 1. *The firm faces:*

1. *An "In" Demand Curve if* $(1 - \phi)Q^{t-1} + \phi b \geq Q_H$.
2. *An "Out" Demand Curve if* $(1 - \phi)Q^{t-1} + \phi b \leq Q_L$.
3. *A "Between" Demand Curve otherwise.*

Given that influencers can help firms become — or stay — "in," firms may be willing to pay influencers for their services. An "out" firm can become "in" without help from an influencer by dropping its price below p_{min} for one period; but this is expensive. With help from an influencer, an "out" firm can become "in" without dropping its price as much. In a competitive setting, an "in" firm might also pay an influencer in order to protect its "in" position against rivals (see Section 3 for further discussion of the competitive case).

In practice, one way in which an influencer might operate is by changing default options. Defaults appear to have the ability to alter consumer expectations — and/or act as nudges — and thereby affect initial impulses.

An illustration can be seen in the "browser war" in the 90s between the originally-dominant Netscape and Microsoft's Internet Explorer. After initial difficulties penetrating the market, Internet Explorer eventually managed to displace Netscape; and when this happened, it happened suddenly.[17] The key to overtaking Netscape was a deal between Microsoft and the internet provider AOL, whereby AOL agreed to

[17] In the mid 1990s, Netscape controlled roughly 80% of the market; by the early 2000s, Internet Explorer controlled more than 90%.

set Internet Explorer as its default browser in exchange for valuable advertising. As Yoffie and Cusumano (1998) note: "To entice Steve Case, the CEO of AOL, to make Internet Explorer AOL's preferred browser, Gates offered to put an AOL icon on the Windows 95 desktop, perhaps the most expensive real estate in the world. In exchange for promoting Internet Explorer as its default browser, AOL would have almost equal importance with [AOL's rival] MSN on future versions of Windows." To this day, the browser wars continue, with smartphones being the latest battlefront. Here again, defaults appear to play a major role (e.g. Cain Miller (2012)).

Remark: Large Consumers

Suppose, in addition to small consumers, there is a large consumer who has the ability to purchase a large quantity of the monopolist's good. Like an influencer, a large consumer can help tip the monopolist from "out" to "in." Consequently, one would expect the monopolist to pay the large consumer a rent — just as he would pay a rent to an influencer. This rent might take the form of a discount relative to the price charged to small consumers.

We see such rents in the music streaming business, for instance. Services like Spotify, Apple Music, and Tidal involve large network externalities.[18] It is not surprising, then, that both Apple Music and Tidal sought to challenge Spotify's dominant position by signing big-name artists such as Beyonce, Drake, Frank Ocean and Kanye West, to exclusive deals on favorable terms to the artists. According to *Rolling Stone*, "Superstar exclusives...have helped Apple and, to a lesser extent, Tidal generate millions of new customers, intensifying competition with Spotify."[19]

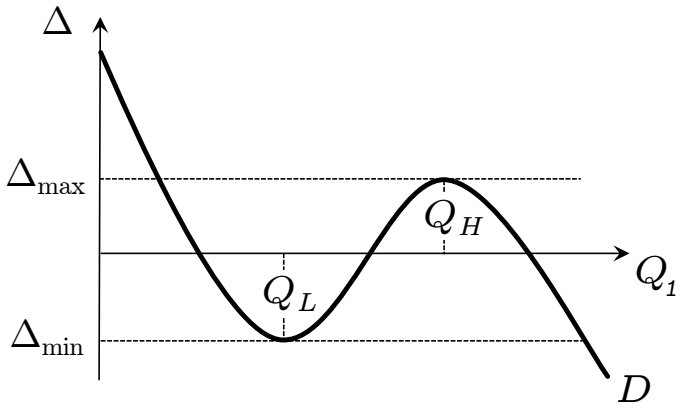
3 Competition

It is a simple step to move from a monopoly setting to a competitive setting. Suppose there are two firms (1 and 2) that engage in price competition. For brevity, we assume

[18] In this case, there are cross-good network externalities (see Section 5 for a formal discussion). An increase in the number of artists on a service makes it more attractive to users; a larger user base, in turn, increases the willingness of artists to join a service.

[19] Steve Knopper, "How Apple Music, Tidal Exclusives Are Reshaping Music Industry," Rolling Stone, October 5, 2016, retrieved from `http://www.rollingstone.com`.

Figure 6: In/Out Demand Curve: Price Competition



there is a single period of competition (but our results readily extend to the multi-period case). There are two stages: in stage 1, firm 1 sets price p_1; in stage 2, firm 2 sets price p_2.

We continue to assume there is a continuum of consumers $i \in [0, 1]$ with tastes θ_i distributed F. Now, θ_i represents consumer i's taste for good 1 *relative to* good 2. Consumers make a binary choice whether to consume good 1 or good 2 — that is, the market is fully covered. Hence, overall demand, $Q_1 + Q_2$, sums to 1. The utility from consuming good 1 is $\theta_i + \mu + \alpha \cdot Q_1 - p_1$, where μ denotes the quality of good 1 relative to good 2. The utility from consuming good 2 is $\alpha \cdot Q_2 - p_2$.

Under these assumptions, demand for each good depends upon the price differential: $\Delta = p_1 - p_2$. Inverse demand for firm 1 is given by:

$$\Delta^d(Q_1) = \mu + \alpha \cdot (2Q_1 - 1) + F^{-1}(1 - Q_1). \tag{7}$$

We will focus on the case where demand is in/out, as pictured in Figure 6. Observe that, whenever demand for good 1 is in/out, demand for good 2 will also be in/out (given that $Q_2 = 1 - Q_1$).[20]

Remark: Product Compatibility

[20]Were we to plot the in/out demand curve for good 2, we would place $p_2 - p_1 = -\Delta$ on the y-axis rather than Δ.

It is easy to incorporate the idea that competing products may be more or less compatible.[21] Suppose the utility from consuming good 1 is $\theta_i+\mu+\alpha\cdot(Q_1+\gamma Q_2)-p_1$ and the utility from consuming good 2 is $\alpha\cdot(Q_2+\gamma Q_1)-p_2$. The parameter $\gamma \in [0,1]$ represents the degree of compatibility of the goods; when they are more compatible, good 1 consumers derive more utility from the consumption of good 2 (and vice-versa). The baseline model has perfect incompatibility ($\gamma = 0$).

This addition to the model has the following effect on the threshold for consuming good 1 rather than good 2:

$$\hat{\theta} = \Delta - \mu - \underbrace{\alpha(1-\gamma)}_{\text{"effective network parameter"}}\cdot(2Q_1 - 1). \tag{8}$$

The only change relative to the baseline model is that $\alpha(1-\gamma)$ appears in place of α. Hence, greater product compatibility (higher γ) is equivalent, from the point of view of the firms, to smaller network externalities (lower α).

3.1 Equilibrium Selection

Analogous to the single-firm case, when the price differential Δ is in an intermediate range, there are multiple Nash equilibria ($Q_1^{out}(\Delta)$, $Q_1^{mid}(\Delta)$, and $Q_1^{in}(\Delta)$). To select between them, we will use the same equilibrium refinement as before. Once again, firm 1 faces one of three negatively-sloped demand curves, as shown in Figure 7.

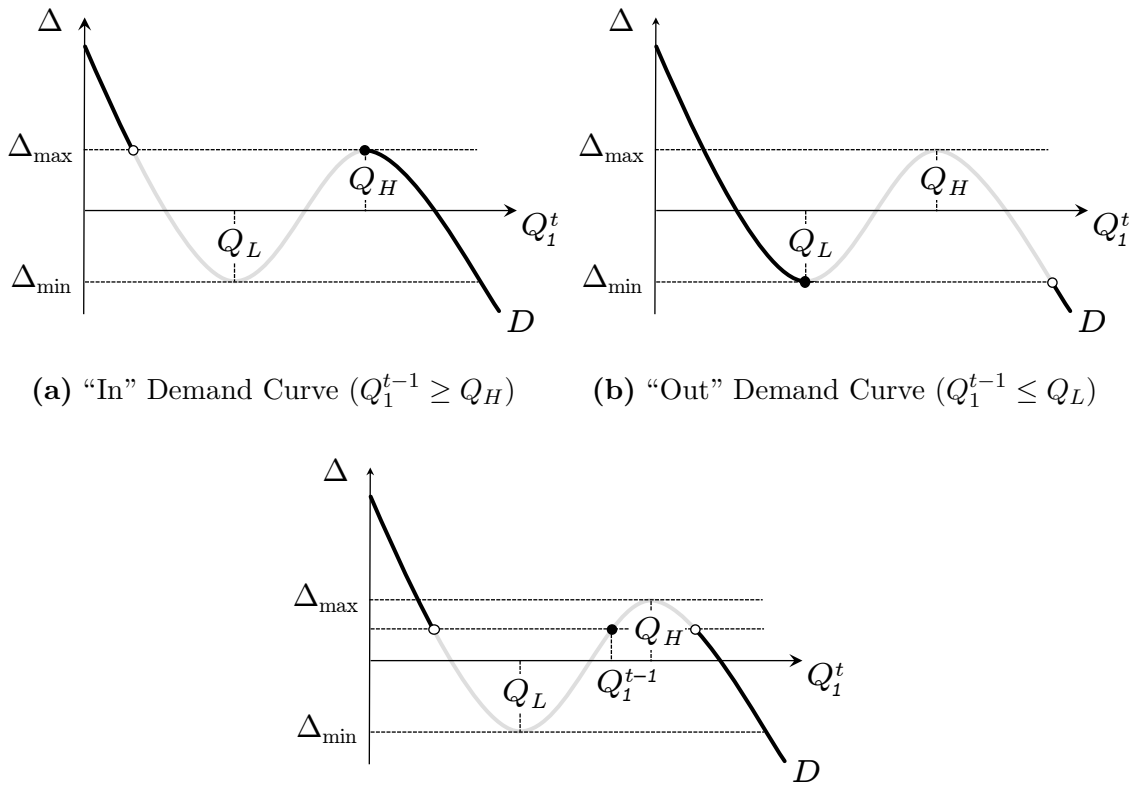
Proposition 4. *In any period, firm 1 faces one of three downward-sloping demand curves (depending upon Q_1^{t-1}):*

1. *"In" Demand Curve ($Q_1^{t-1} \geq Q_H$): Figure 7a.*
2. *"Out" Demand Curve ($Q_1^{t-1} \leq Q_L$): Figure 7b.*
3. *"Between" Demand Curve ($Q_L < Q_1^{t-1} < Q_H$): Figure 7c.*[22]

[21] The existing literature on network externalities has highlighted product compatibility as an issue of interest: particularly, the incentives of firms to make their products compatible (see, for instance, Katz and Shapiro (1994)).

[22] From the demand curve for firm 1, it is easy to derive the demand curve for firm 2, as $Q_2 = 1 - Q_1$. Note that firm 2 is "in" when $Q_2^{t-1} \geq 1 - Q_L$, firm 2 is "out" when $Q_2^{t-1} \leq 1 - Q_H$, and firm 2 is "between" when $1 - Q_H < Q_2^{t-1} < 1 - Q_L$.

Figure 7: Price Competition



(a) "In" Demand Curve ($Q_1^{t-1} \geq Q_H$)

(b) "Out" Demand Curve ($Q_1^{t-1} \leq Q_L$)

(c) "Between" Demand Curve ($Q_L < Q_1^{t-1} < Q_H$)

We will again refer to a firm as "in," "out," or "between" depending upon whether it faces an "in," "out," or "between" demand curve. Because overall demand is fixed ($Q_1 + Q_2 = 1$), either both firms are "between" or one is "in" and the other is "out."

We will focus attention in what follows on the case where firm 1 starts "in" and firm 2 starts "out." This corresponds to many cases of interest where competition is between an established firm that has built up a network and a recent entrant.

3.2 Analysis

Recall that, in stage 1, firm 1 sets a price p_1; in stage 2, firm 2 sets a price p_2. The resulting payoffs to the firms are $\pi_1 = p_1 \cdot Q_1^{in}(\Delta)$ and $\pi_2 = p_2 \cdot (1 - Q_1^{in}(\Delta))$, where $\Delta = p_1 - p_2$. Observe that π_1 and π_2 depend upon the shape of the demand curve;

the demand curve, in turn, depends upon parameters α, μ, and F.

We can use backward induction to solve for the equilibrium of this game. Let $p_2^{BR}(p_1)$ denote firm 2's best response to price p_1 and let $\Delta(p_1) = p_1 - p_2^{BR}(p_1)$. Firm 1 chooses p_1 to maximize:

$$\pi_1(p_1) = p_1 \cdot Q_1^{in}(\Delta(p_1)).$$

Firm 1 "remains in" if $\Delta(p_1) \leq p_{\max}$ and "falls out" if $\Delta(p_1) > p_{\max}$. We will refer to $\Delta(p_1) \leq p_{\max}$ as the "remain-in constraint" or RIC. This constraint will bind whenever (1) firm 1's strategy is to remain in, and (2) the quality differential between the firms is not too large, so competition for the "in" position is intense.

It is easy to show that, to remain in, firm 1 must set a price below a threshold $\overline{p}_{\text{RIC}}$ (the formal argument is given as part of the proof of Proposition 5 below). Therefore, the following is an equivalent formulation of the remain-in constraint:

$$p_1 \leq \overline{p}_{\text{RIC}}(\alpha, \mu, F). \tag{RIC}$$

Proposition 5 characterizes how a change in the goods' relative qualities (μ) affects the equilibrium outcome in the region where RIC binds.

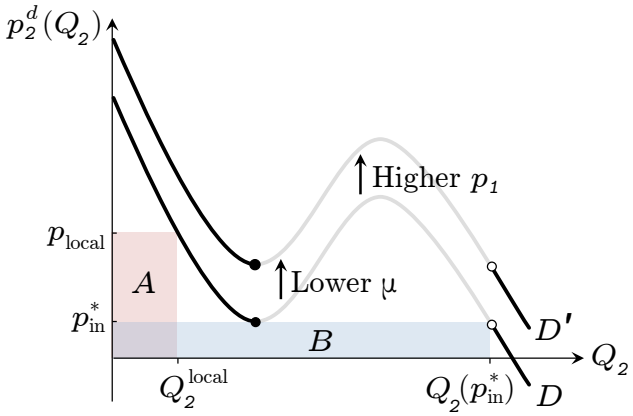
Proposition 5. *When RIC binds, increases in the quality of good 1 relative to good 2, as measured by μ:*

1. *Translate one-to-one into increases in good 1's equilibrium price: $\frac{\partial \overline{p}_{RIC}}{\partial \mu} = 1$.*
2. *Have no effect on good 2's equilibrium price.*
3. *Have no effect on equilibrium quantities (Q_1 and Q_2).*

Proof of Proposition 5. Figure 8 shows the demand for good 2 for a particular value of p_1. Firm 2's best response to p_1 is either to choose the profit-maximizing price conditional on staying "out" (p_{local} in the figure), or the profit-maximizing price conditional on going "in" (p_{in}^* in the figure).[23]

[23] In the figure, p_{in}^* is depicted as being right at the threshold for firm 2 to go "in" but p_{in}^* may also be lower than that threshold.

Figure 8: The Remain-In Constraint (RIC)



Region A in Figure 8 represents the profits to firm 2 from choosing p_{local}; Region B represents the profits from choosing p^*_{in}. Observe that RIC is satisfied when Region A is (weakly) larger than Region B; RIC binds when the regions are of equal size.

An increase in p_1 shifts firm 2's demand curve vertically up, which increases the size of Region B relative to Region A.[24] This explains why firm 1 must price below a threshold, $\overline{p}_{\text{RIC}}$, in order to meet RIC.

Suppose RIC is a binding constraint and suppose demand curve D in Figure 8 depicts a binding RIC. Observe that the demand curve firm 2 faces depends upon the "effective price" of good 1: $p_1 - \mu$. Hence, if μ decreases by an amount $\Delta\mu$, firm 1 must decrease p_1 by $\Delta\mu$ to stay on demand curve D. This explains why, in the region where RIC binds, a change in μ changes p_1 by an equivalent amount. Furthermore, since firm 2 always faces the same demand curve D in the region where RIC binds, it always charges the same price (p_{local}) and sells the same quantity (Q_2^{local}). QED.

In practice, "in" firms may need to charge low — even zero — prices to satisfy the RIC constraint. For example, despite their overwhelming market shares, Google (in web search), Uber (in ride sharing), and Amazon Web Services (in cloud computing) all keep their prices low — arguably to stunt the rise of their nearest rivals.[25]

In the case of Uber, its inability to charge higher prices has led to significant

[24] This observation follows from the Envelope Theorem and the fact that $Q_2(p^*_{in}) > Q_2^{local}$.

[25] Note that one way in which online firms may charge users is by showing them advertisements.

losses. As noted in Forbes: "Uber made a calculated bet that it could achieve global domination, wiping out both incumbent taxi companies and competing shared ride providers, to be able to exercise monopoly pricing power in hundreds of metropolitan markets. But it now appears Uber has lost this bet."[26] Our model helps explains this failure. Regardless of its market share, a low degree of differentiation with its rivals (a low μ) means that Uber can only meet its RIC constraint by means of low prices, making it difficult (perhaps even impossible) to recoup its substantial early investments.

Note that the "out" firm's check on the "in" firm is a generalized form of "limit pricing," whereby a monopolist is disciplined by a potential entrant. In our case, rather than deterring entry outright, the winning firm needs to deter the losing firm from becoming popular. It does so by allowing the losing firm to enjoy rents from a small but loyal consumer base, a form of consolation prize.

3.3 Incentives for Innovation

The fact that when network externalities are large, the winning and losing firms compete for the "in" position, as opposed to merely competing for a single marginal consumer, has significant implications for the firms' incentives to innovate.

To illustrate, suppose the two firms have an opportunity to invest up front on R&D activities that raise the intrinsic quality of their respective products. Let μ_i denote the intrinsic quality of firm's i product, with $\mu = \mu_1 - \mu_2$. Suppose quality μ_i costs $C(\mu_i)$ to obtain, where C is twice differentiable and satisfies $C', C'' > 0$ and $C'(0) = 0$. Suppose μ_1 and μ_2 are observed by both firms before they engage in price competition. (The exact timing of the choices of μ_1 and μ_2 is immaterial.)

Corollary 2 shows that, when the network externality is strong, the two firms face radically different incentives to innovate:

Corollary 2. *Consider the extended model with investments. Suppose firm 1 retains the "in" position and suppose that, in the pricing stage, the remain-in constraint is binding. Then:*

[26] See Len Sherman, "Why Can't Uber Make Money?," Forbes, December 14, 2017, retrieved from `http://www.forbes.com`.

1. *Firm 1's optimal investment* μ_1^* *satisfies* $C'(\mu_1^*) = Q_1^*$*, where* Q_1^* *denotes the equilibrium sales of firm* 1.

2. *Firm 2 has zero incentive to innovate.*

This result follows from the fact that when the remain-in constraint is binding, we have $\frac{dp_1}{d\mu} = 1$. As a result, an increase in firm 1's quality translates one-to-one into an *increase* in its equilibrium price, and hence firm 1 invests in direct proportion to the size of its own market (which, given its winning position, is large). In contrast, an increase in firm 2's quality translates one-to-one into an *reduction* in its rival's price; thus, this higher quality has zero impact on firm 2's revenues.

In practice, firms may also increase the quality of their products by acquiring startups with valuable product innovations. Indeed, such acquisitions are commonplace. In the decade between 2008 and 2017, Google/Alphabet made 166 acquisitions, Amazon 51, Facebook 63, Ebay 31, Twitter 54, and Apple 66.[27]

Competition for the "in" position may lead to highly asymmetric outcomes. To illustrate, suppose a third party (a "startup") possesses an innovation and, prior to engaging in price competition, firms 1 and 2 bid in a (second-price) auction to buy the startup. Suppose the firm that acquires the startup adopts its innovation, and as a result improves its quality by $\Delta\mu$.

In this setting, provided the hypothesis of Corollary 2 is met, firm 1's maximum bid for the startup is $2\Delta\mu Q_1$; whereas firm 2's maximum bid is 0.[28] Therefore, firm 1 acquires the startup and pays 0 for it, further cementing it dominant position. In fact, in a multi-period version of this merger game in which a new startup emerges in every period, the dominant firm outbids its rival for each new startup; thus, its dominant position becomes more and more entrenched as time goes by.

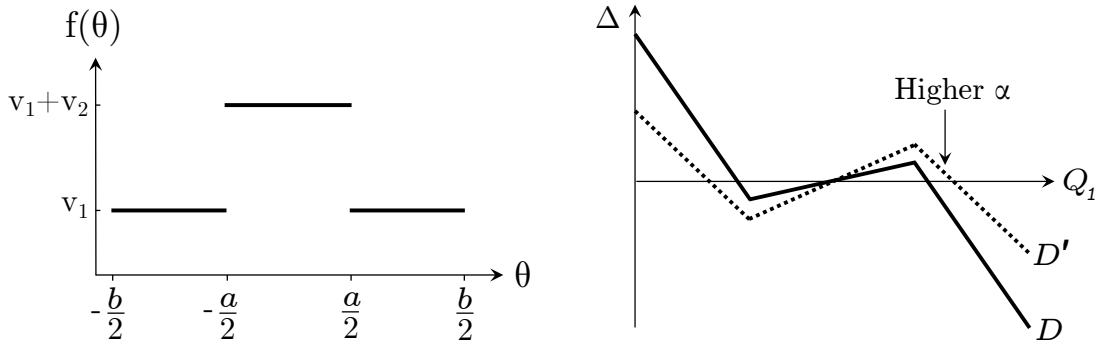
[27]Consider a few of Apple's acquisitions. PA Semi (purchased in 2008 for $278 million): a California-based chip designer whose acquisition was instrumental to Apple's development of low-power processors. Siri (purchased in 2010 for $250 million): this virtual personal assistant technology has been integrated into a variety of Apple devices. C3 Technologies (purchased in 2011 for $273 million): one of several startups acquired by Apple to improve its mapping features. PrimeSense (purchased in 2013 for $360 million): an Israeli 3D sensing company whose technology powers the facial recognition features of the iPhone X.

[28]By winning, firm 1 not only increases its quality by $\Delta\mu$, it also prevents firm 2 from increasing its quality by $\Delta\mu$. Hence firm 1 is willing to bid $2\Delta\mu$ per unit of expected sales.

4 Piecewise Linear Demand

Suppose firms 1 and 2 are engaged in the price competition game described in the previous section. Suppose further that consumers' tastes follow the step distribution depicted in Figure 9a. In this case, the demand curves for firms 1 and 2 are piecewise linear, as depicted in Figure 9b. Piecewise linear demand allows us to solve explicitly for the outcome of price competition.[29]

Figure 9: Piecewise Linear Demand



(a) Pdf. **(b)** Corresponding demand curve.

We restrict attention to the case where α lies between $\alpha_{\min} = \frac{1}{2(v_1+v_2)}$ and $\alpha_{\max} = \frac{1+a(v_1-v_2)}{2v_1(1+a(v_1+v_2))}$. The condition $\alpha > \alpha_{\min}$ ensures demand is in/out; the condition $\alpha < \alpha_{\max}$ ensures demand is positive for $p > p_{\max}$.

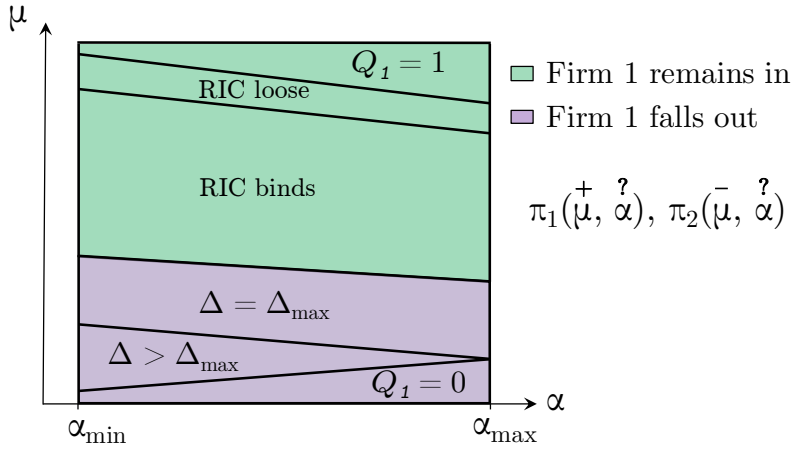
As Figure 10 shows, the equilibrium of the game depends upon the region into which the parameters μ and α fall (this figure is drawn to scale for the case where $a = 1$ and $v_1 = v_2 = \frac{1}{3}$).[30]

Several points are worth making. First, as one would expect, firm 1 "remains in" if μ (good 1's relative quality) is above a threshold; firm 1 "falls out" if μ is below the threshold. The remain-in constraint (RIC) is binding when μ is just above the threshold. When μ is just below the threshold, firm 2 chooses the minimal price that puts it "in" ($\Delta = \Delta_{\max}$).

[29] Piecewise linear demand also facilitates an analysis of the effects of demand volatility. See the Appendix for further discussion.

[30] Appendix B specifies the equilibrium prices and quantities in each region.

Figure 10: Equilibrium Outcome with Piecewise Linear Demand



Second, as one crosses the threshold from the region where RIC is satisfied to the region where RIC is violated, prices jump discontinuously. Firm 1's price jumps up and firm 2's price jumps down. Intuitively, prices jump because firm 1 gives up on remaining "in" and firm 2 decides it is worthwhile to go "in."

Third, competition between the firms is "normal" in the regions where RIC is loose and where $\Delta > \Delta_{\max}$. Here, changes in relative quality, μ, impact the prices and quantities of both goods. Competition is abnormal when RIC binds or $\Delta = \Delta_{\max}$. In those regions, changes in μ have no effect on quantities and only affect firm 1's price.

Finally, one might think that the firm that ends up "in" benefits from an increase in network externalities (α). In fact, it is ambiguous whether an increase in α benefits or hurts the "in" firm. The reason is that an increase in α has two effects (which are illustrated in Figure 9b):

1. For a given price differential, the "in" firm gets a larger share of the market.

2. Demand is more elastic.

The "in" firm benefits from the first effect. The second effect, however, can drive more intense competition between the firms for the "in" position. This competition may be harmful to the "in" firm. A lesson is that, even in cases where one firm has

a dominant market share, it may be incorrect to assume that competition is weak.

5 Multiple Goods and Platforms

It is easy to extend our model to the case of a multi-sided platform, where a firm sells multiple goods and there are cross-good externalities. One example is Uber, which sells two goods: passenger rides and driver rides. Passengers care about the number of drivers; drivers, similarly, care about the number of passengers.

Suppose the monopolist sells good 1 to one population and good 2 to a separate population. Consumers in population j are potential consumers of good j. Each consumer has a type θ; the θ's are distributed F_j for good j. Consumer i's utility from consuming good j is:

$$\theta_i + \mu_j + \alpha_j \cdot Q_j + \beta_j \cdot Q_l - p_j,$$

where p_j denotes the price of good j, μ_j denotes the intrinsic quality of good j, the term $\alpha_j \cdot Q_j$ represents a same-good network externality, and the term $\beta_j \cdot Q_l$ represents a cross-good network externality. As before, we normalize to zero the utility from not consuming.

The following is an analog of equation (3):

$$Q_j = 1 - F_j(p_j - \mu_j - \alpha_j \cdot Q_j - \beta_j \cdot Q_l). \tag{9}$$

From equation (9), we obtain a formula for (inverse) demand:

$$p_j^D(Q_j, Q_l) = F_j^{-1}(1 - Q_j) + \alpha_j \cdot Q_j + \beta_j \cdot Q_l + \mu_j. \tag{10}$$

To illustrate, consider the simple "symmetric" case where $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, and $F_1 = F_2$. In addition, suppose the two goods have equal initial impulses: $Q_1^0 = Q_2^0 = Q^0$. Provided a suitable concavity assumption is satisfied, the firm sets a single price: $p_1 = p_2 = p$. In this case, the model is isomorphic to the baseline single-good monopoly environment, with $\alpha + \beta$ appearing in place of α.

As noted in the literature, there is often asymmetric pricing ($p_1 \neq p_2$) in two-sided markets (see especially Rochet and Tirole (2003)). Such pricing is optimal, for example, when the cross-network externalities are asymmetric ($\beta_1 \neq \beta_2$). We leave this extension for future work.

This framework is also amenable to analyzing platform competition. Here, there is an analog to our results for the single-good case: it is difficult to go from "out" to "in" and yet the "in" position is fragile; firms rely on influencers; and winning firms, despite their dominant market shares, charge low prices and seek to entrench their positions via sustained quality investments.

6 Conclusion

We proposed a rich yet tractable framework to study optimal pricing and price competition in the presence of network effects.

A critical feature of markets with networks externalities is their ability to generate multiple equilibria. Such multiplicity is behind large asymmetries between winners and losers. It is also behind the apparent paradox that it is difficult to become a winner, and yet the winning position is fragile; hence, winners are not asleep.

Understanding how one equilibrium gets picked over another is essential. We proposed a simple theory of equilibrium selection that captures the notion that a firm's popularity exhibits a form of inertia over time, and is affected as well by salient consumers that are popular among their peers. A firm's default popularity, inherited from the previous period, then determines whether it currently faces its worst possible demand curve (the "out" demand), its best possible curve (the "in" demand), or an intermediate version of the two (the "between" demand). Each of these demand curves has a well-behaved shape with a standard negative slope, but with a discontinuity. This simple classification immediately sheds light on the firm's optimal pricing, its equilibrium transitions between the losing and the winning positions, and its incentives for innovation.

Our model features a form of asymmetric competition in which winning and losing firms co-exist, with the losing firm keeping the winning firm in check. This check on

the winning firm is a generalized form of "limit pricing," whereby a monopolist is disciplined by a potential entrant. In our case, rather than deterring entry outright, the winning firm needs to deter the losing firm from becoming popular. It does so by allowing the losing firm to enjoy rents from a small but loyal consumer base, a form of consolation prize.

For the interested reader, Appendix A analyzes a simple version of the model where all consumers are of the same type. This version gives the starkest case where impulses play a role; but it lacks the richness of the in/out demand curve. In addition, Appendix B analyzes the problem of a monopolist who faces volatile demand. In this case, the monopolist lowers its price to protect against the risk of falling "out."

In subsequent work, we expect to propose a method for valuing firms in the presence of network effects, with such effects opening the possibility of large profits for popular firms, but also leading to the risk of sudden failure.

Appendix

A A Simple Case: Homogeneous Consumers

Here, we analyze a simple version of the model where all consumers are of the same type, and therefore aggregate demand is either 0 or 1. This version gives the starkest case where impulses play a role.

We focus attention on a monopoly setting.[31] Under the assumption that all consumers are of type $\theta = 0$, demand in period t is given by:

$$Q^t\left(p^t, Q^{t-1}\right) = \begin{cases} 1, & \text{if } p^t \leq \mu + \alpha Q^{t-1}. \\ \\ 0, & \text{if } p^t > \mu + \alpha Q^{t-1}. \end{cases}$$

The monopolist faces an "in" demand curve in period t if $Q^{t-1} = 1$, an "out" demand curve in period t if $Q^{t-1} = 0$, or a "between" demand curve in period t if $0 < Q^{t-1} < 1$.

Suppose, as in Section 2, that the monopolist chooses a price in each of T periods and has a discount factor of δ. Let us solve for the optimal choice of prices.

From Proposition 3, it follows that we can restrict attention to two possible pricing strategies. Strategy 1: go "in" in period 1 and stay "in" in subsequent periods. Strategy 2: go "out" in period 1 and stay "out" in subsequent periods.

Strategy 1 involves setting a price $p^1 = \mu + \alpha Q^0$ in the first period and a price $p^t = \mu + \alpha$ in subsequent periods ($t > 1$). The profits associated with this strategy are: $\sum_{t=1}^{T} \delta^{t-1} p^t = \mu + \alpha\left(Q^0 + \frac{\delta - \delta^T}{1-\delta}\right)$. Strategy 2 yields a payoff of 0 to the monopolist.

The monopolist will follow Strategy 1 if and only if it yields nonnegative profits, or:

$$\mu \geq -\alpha\left(Q^0 + \frac{\delta - \delta^T}{1-\delta}\right)$$

[31] When there is competition, the model becomes very similar to Halaburda et al. (2018)'s. The reason is that, when consumers are homogeneous, our focality criterion coincides with theirs.

If the monopolist's good is of sufficiently high quality ($\mu \geq -\alpha\left(\frac{\delta-\delta^T}{1-\delta}\right)$), the monopolist chooses to go "in" (i.e., follow Strategy 1) regardless of consumers' initial impulse. Similarly, if the monopolist's good is of sufficiently low quality $\mu < -\alpha\left(1+\frac{\delta-\delta^T}{1-\delta}\right)$, the monopolist chooses to go "out" (i.e., follow Strategy 2) regardless of consumers' initial impulse.

If the quality of the monopolist's good is in an intermediate range ($-\alpha\left(\frac{\delta-\delta^T}{1-\delta}\right) > \mu \geq -\alpha\left(1+\frac{\delta-\delta^T}{1-\delta}\right)$), the monopolist's strategy depends upon the impulse. The monopolist chooses to go "in" (follow Strategy 1) if and only if the initial impulse to consume is above a threshold: $Q^0 \geq -\left(\frac{\mu}{\alpha}+\frac{\delta-\delta^T}{1-\delta}\right)$.

Observe that an increase in the number of time periods (T) makes the monopolist more inclined to go "in" (i.e., it decreases the threshold quality for following Strategy 1). This is intuitive since the monopolist will be more willing to pay an initial cost of going "in" when there are more subsequent periods in which to reap the rewards.

B Demand Volatility

Consider the problem of a monopolist who prices for a single period and faces a random μ. Suppose that, for any given μ, demand is in/out and piecewise linear.[32]

We assume that $\mu = \hat{\mu} + \varepsilon$, where:

$$\varepsilon = \begin{cases} \sigma, & \text{with probability } r. \\ -\sigma, & \text{with probability } r. \\ 0 & \text{with probability } 1-2r. \end{cases}$$

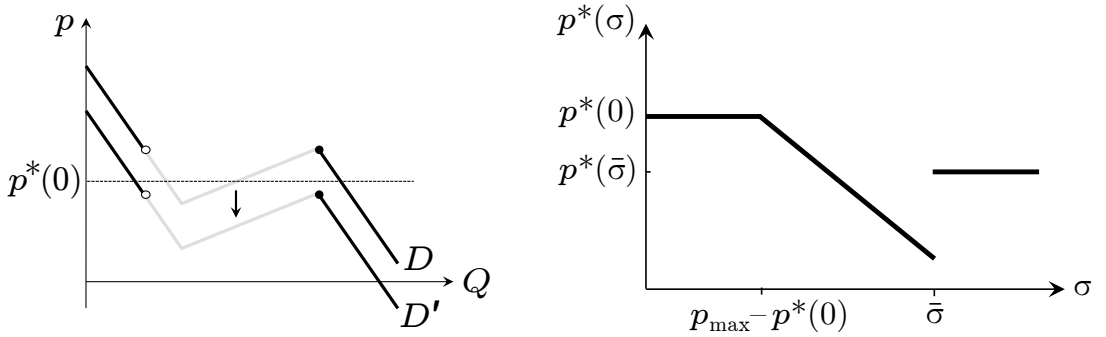
The resulting demand curve can be expressed as:

$$p^d(Q) = \hat{p}(Q) + \varepsilon,$$

where $\hat{p}(Q)$ is in/out and piecewise linear (see Figure 11a).

[32]That is, suppose consumers' tastes are distributed as in Figure 9a and network externalities are sufficiently large ($\alpha > \frac{1}{v_1+v_2}$).

Figure 11: Optimal Pricing when Demand is Volatile



(a) There is risk of going from "in" to "out."

(b) Optimal Price.

How does demand volatility affect optimal pricing? Let $p^*(\sigma)$ denote the optimal price for a given level of volatility, σ. Let us focus attention on the case where the monopolist is "in" for sure when there is no demand volatility ($\sigma = 0$). Figure 11a illustrates that, if demand is sufficiently volatile, the firm risks going "out" if it keeps its price at $p^*(0)$. Going "out" is quite costly as it involves a discontinuous decline in demand. Therefore, the firm has an incentive to shade its price.

Figure 11b shows the optimal price as a function of the volatility. When volatility is low ($\sigma \leq p_{\max} - p^*(0)$), the monopolist does not risk going out if it sets a price $p^*(0)$. Consequently, the optimal price is simply $p^*(0)$. When volatility is in an intermediate range ($p_{\max} - p^*(0) < \sigma < \overline{\sigma}$), the firm shades its price to eliminate the risk of going "out." In this region $p^*(\sigma) = p^*(0) - [\sigma - (p_{\max} - p^*(0))]$. When volatility is sufficiently high ($\sigma > \overline{\sigma}$), the cost of shading is sufficiently high that the firm chooses to accept some risk of going "out." In this region, the firm charges a price $p^*(\overline{\sigma})$ and it goes "out" with probability r.[33]

C Competition when Demand is Piecewise-Linear

Consider the pricing game described in Section 3 and suppose demand is piecewise-linear (as in Figure 9). There are six regions into which the parameters μ and α

[33]The reason $p^*(\overline{\sigma}) < p^*(0)$ is as follows. While the demand curve has the same slope when the firm is "out" as it does when the firm is "in," demand is actually more elastic in the "out" region. Hence, the price markup is smaller when there is a chance of going "out."

can fall, ordered from a highest-μ region to a lowest-μ region. The outcome for each region is as follows (see the Online Supplement for derivations):[34]

1. RIC is loose and $Q_1^* = 1$ ($\mu \geq \overline{\mu}_1$):

$$p_1^* = \mu + \alpha + \frac{-1 + av_2}{2v_1},$$
$$p_2^* = 0.$$

2. RIC is loose and $Q_1^* < 1$ ($\overline{\mu}_1 > \mu \geq \overline{\mu}_2$):

$$p_1^* = \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 + av_2}{4v_1},$$
$$p_2^* = -\frac{1}{4}\mu - \frac{5}{4}\alpha + \frac{5 - av_2}{8v_1},$$
$$Q_1^* = \frac{v_1}{2(1 - 2v_1\alpha)}(\frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 + av_2}{4v_1}).$$

[34]The Online Supplement is available at: `http://www.robertakerlof.com/research.html`.

3. RIC binds ($\overline{\mu}_2 > \mu \geq \overline{\mu}_3$):

$$p_1^* = \mu + \alpha(2a(v_1+v_2)-1) + \frac{a(3v_2-2v_1)+1}{2v_1} - \frac{2}{v_1}\sqrt{av_2[\frac{1}{2}(1+a(v_2-v_1)) - \alpha v_1(1-a(v_1+v_2))]},$$

$$p_2^* = \alpha(a(v_1+v_2)-1) + \frac{a(v_2-2v_1)+3}{4v_1} - \frac{1}{v_1}\sqrt{av_2[\frac{1}{2}(1+a(v_2-v_1)) - \alpha v_1(1-a(v_1+v_2))]},$$

$$Q_1^* = \frac{1-a(v_2-v_1)}{2(1-2v_1\alpha)} - \frac{\alpha v_1(a(v_1+v_2)+1)}{1-2v_1\alpha} + \frac{1}{1-2v_1\alpha}\sqrt{av_2[\frac{1}{2}(1+a(v_2-v_1)) - \alpha v_1(1-a(v_1+v_2))]}.$$

4. $\Delta = \Delta^+_{\max}$ ($\overline{\mu}_3 > \mu \geq \overline{\mu}_4$):

$$p_1^* = \mu + \alpha(2a(v_1+v_2)-1) + \frac{1+a(v_2-2v_1)}{2v_1},$$
$$p_2^* = \alpha(a(v_1+v_2)-1) + \frac{1+a(v_2-v_1)}{2v_1},$$
$$Q_1^* = \frac{1-a(v_2-v_1)-2v_1\alpha(1+a(v_1+v_2))}{2(1-2v_1\alpha)}.$$

5. $\Delta > \Delta^+_{\max}$ and $Q_1^* > 0$ ($\overline{\mu}_4 > \mu \geq \overline{\mu}_5$):

$$p_1^* = \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3-av_2}{4v_1},$$
$$p_2^* = -\frac{1}{4}\mu - \frac{5}{4}\alpha + \frac{5+av_2}{8v_1},$$
$$Q_1^* = \frac{v_1}{2(1-2v_1\alpha)}(\frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3-av_2}{4v_1}).$$

6. $\Delta > \Delta^{+}_{\max}$ and $Q^{*}_{1} = 0$ ($\mu < \overline{\mu}_5$):

$$p^{*}_{1} = 0,$$
$$p^{*}_{2} = -\mu + \alpha - \frac{1 - av_2}{2v_1}.$$

The cutoffs between regions are:

$$\overline{\mu}_1 = \frac{5 - av_2}{2v_1} - 5\alpha,$$

$$\overline{\mu}_2 = \frac{1 - a(5v_2 - 4v_1)}{2v_1} - \alpha(4a(v_1 + v_2) + 1) + \frac{4}{v_1}\sqrt{av_2[\frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2))]},$$

$$\overline{\mu}_3 = \frac{1 - a(5v_2 - 4v_1)}{2v_1} - \alpha(1 + 4a(v_1 + v_2)) + \frac{1 + 3a(v_2 - v_1) - 2\alpha v_1(1 - 3a(v_1 + v_2))}{v_1[1 + a(v_2 - v_1) + 2\alpha v_1(-1 + a(v_1 + v_2))]} \times \sqrt{av_2[\frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2))]},$$

$$\overline{\mu}_4 = \frac{1 - a(3v_2 - 4v_1)}{2v_1} - \alpha(4a(v_1 + v_2) + 1),$$

$$\overline{\mu}_5 = \frac{-3 + av_2}{2v_1} + 3\alpha.$$

D Proofs

Proof of Proposition 1.

Let $p_{\min} \leq p \leq p_{\max}$. First, $Q^{t-1} > Q_H$ implies $Q^{t-1} > Q^{mid}(p)$; therefore, from Lemma 1, the firm faces an "in" demand curve. Second, $Q^{t-1} = Q_H$ implies that either $p < p_{\max}$ and $Q^{t-1} > Q^{mid}(p)$, or $p = p_{\max}$ and $Q^{t-1} = Q^{mid}(p) = Q^{in}(p)$; therefore, from Lemma 1, the firm faces an "in" demand curve. Third, when $Q^{t-1} \leq Q_L$, a similar argument implies that the firm faces an "out" demand curve. Finally, $Q_H > Q^{t-1} > Q_L$ implies that Q^{t-1} is greater than (respectively, equal to or less than) $Q^{mid}(p)$ if p is less than (respectively, equal to or greater than) the threshold $(Q^{mid})^{-1}(Q^{t-1})$; therefore, from Lemma 1, the firm faces a "between" demand curve.

Proof of Proposition 2.

When $\delta = 0$, the firm sets p^t to maximize $\pi^t = p^t \cdot Q^*(p^t, Q^{t-1})$. Lemma A1 (stated below) says that, when the firm prices optimally, it ends period t either "in" or "out" — not "between."

Lemma A1. *If the firm is myopic ($\delta = 0$) and prices optimally, it ends period t either "in" ($Q^t \geq Q_H$) or "out" ($Q^t \leq Q_L$).*

The proof is simple. The only way for the firm to end period t "between" is if it starts the period "between" and chooses a price such that $Q^t = Q^{t-1}$. But, as Figure 4c shows, starting at such a price, a small decrease in the price increases demand by a discontinuous amount. Thus, the hypothesized price cannot be optimal.

Given Lemma A1, we can think of the firm as choosing between: (i) the optimal price conditional on ending period t "in" and (ii) the optimal price conditional on ending period t "out." Let $p^*_{in}(Q^{t-1})$ denote the optimal price conditional on ending period t "in" and let $\pi^*_{in}(Q^{t-1})$ denote the associated profits. Similarly, let $p^*_{out}(Q^{t-1})$ denote the optimal price conditional on ending period t "out" and let $\pi^*_{out}(Q^{t-1})$ denote the associated profits.

To end period t "in," the firm must price below a threshold, $p_{\text{thresh}}(Q^{t-1})$; to end period t "out," the firm must price above this threshold. Observe that, when $Q^{t-1} \leq Q_L$, the threshold is $p_{\min}$; when $Q^{t-1} \geq Q_H$, the threshold is $p_{\max}$; and when

$Q_L < Q^{t-1} < Q_H$, the threshold is between $p_{\min}$ and $p_{\max}$.[35] Observe also that the threshold, $p_{\text{thresh}}(Q^{t-1})$, is weakly increasing in the impulse (Q^{t-1}).

It is easier to end period t "in" when the threshold is higher; and the threshold is higher when the impulse, Q^{t-1}, is higher. Consequently, the "in" price ($p^*_{in}(Q^{t-1})$) and the associated profits ($\pi^*_{in}(Q^{t-1})$) are both weakly increasing in the impulse (Q^{t-1}). This is stated formally as Lemma A2.

Lemma A2. *$p^*_{in}(Q^{t-1})$ and $\pi^*_{in}(Q^{t-1})$ are both weakly increasing in Q^{t-1}.*

Similarly, it is easier to end period t "out" when the threshold is lower. The threshold is lowest when $Q^{t-1} \leq Q_L$ (i.e., when the firm starts period t "out"). Let p_{local} denote the value of $p^*_{out}(Q^{t-1})$ when the threshold is lowest. Let q_{local} denote the associated quantity demanded and let π_{local} denote the associated profits. Observe that π_{local} is an upper bound on $\pi^*_{out}(Q^{t-1})$.

For a given impulse, Q^{t-1}, either: (1) $p_{local} \geq p_{\text{thresh}}(Q^{t-1})$; or (2) $p_{local} < p_{\text{thresh}}(Q^{t-1})$. In case (1), since p_{local} is above the threshold, the "out" price is equal to p_{local} and the "out" profits are equal to π_{local}.

Now consider case (2). Since p_{local} is less than the threshold, when the monopolist sets a price of p_{local}, he ends period t "in" and sells a quantity greater than q_{local}. Since the quantity sold exceeds q_{local}, the monopolist's profits are also greater than π_{local}. Hence, a price of p_{local} results in a profit above π_{local} (the upper bound on "out" profits). It follows that, in case (2), the "in" price yields a higher profit than the "out" price.

We conclude, therefore, that whenever the monopolist chooses the "out" price over the "in" price, the "out" price is equal to p_{local} and the "out" profits are equal to π_{local}. Lemma A3, stated below, summarizes.

Lemma A3. *If the firm chooses to end period t "out," it will set a price of p_{local} and earn a profit of π_{local}.*

Lemmas A2 and A3 imply that, if $\pi^*_{in}(Q^{t-1}) > \pi_{local}$, the firm chooses the "in" price and earns a profit of $\pi^*_{in}(Q^{t-1})$; otherwise, the firm chooses price p_{local} and earns

[35] Formally, the threshold price is the minimum price for which the quantity demanded is greater than or equal to Q_L: $p_{\text{thresh}}(Q^{t-1}) = \min\{p : Q^*(p, Q^{t-1}) \geq Q_L\}$.

a profit of π_{local}. Proposition 2 immediately follows.

Proof of Corollary 1.

This corollary follows directly from Proposition 1 upon redefining the impulse as $(1-\phi)Q^{t-1} + \phi b$.

Proof of Proposition 4.

We begin by stating a lemma.

Lemma A4. *Fix a time t and suppose the aggregate impulse for firm 1 is Q_1^{t-1}. When $\Delta_{\min} \leq \Delta \leq \Delta_{\max}$, the unique introspective equilibrium is:*

$$Q_1^*\left(\Delta, Q_1^{t-1}\right) = \begin{cases} Q_1^{in}(\Delta), & \text{if } Q_1^{t-1} > Q_1^{mid}(\Delta). \\ Q_1^{mid}(\Delta), & \text{if } Q_1^{t-1} = Q_1^{mid}(\Delta). \\ Q_1^{out}(\Delta), & \text{if } Q_1^{t-1} < Q_1^{mid}(\Delta). \end{cases}$$

When $\Delta > \Delta_{\max}$ or $\Delta < \Delta_{\min}$, $Q_1^*\left(\Delta, Q_1^{t-1}\right)$ is the unique solution to equation (9).

The proof of this lemma is very similar to the proof of Lemma 1: the only difference is that Δ takes the place of p. Proposition 4 immediately follows.

Proof of Corollary 2.

This corollary follows directly from Proposition 5.

References

Ambrus, Attila and Rossella Argenziano, "Asymmetric networks in two-sided markets," *American Economic Journal: Microeconomics*, 2009, *1* (1), 17–52.

Argenziano, Rossella and Itzhak Gilboa, "History as a coordination device," *Theory and Decision*, 2012, *73* (4), 501–512.

Armstrong, Mark, "Competition in two-sided markets," *RAND Journal of Economics*, 2006, *37* (3), 668–691.

Arrow, Kenneth J., "The Economic Implications of Learning by Doing," *The Review of Economic Studies*, 1962, *29* (3), 155–173.

Becker, Gary S., "A note on restaurant pricing and other examples of social influences on price," *Journal of Political Economy*, 1991, *99* (5), 1109–1116.

Biglaiser, Gary and Jacques Cremer, "The value of incumbency for heterogeneous platforms," *Working Paper*, 2016.

Brynjolfsson, Erik and Andrew McAfee, *The second machine age: Work, progress, and prosperity in a time of brilliant technologies*, WW Norton & Company, 2014.

Caillaud, Bernard and Bruno Jullien, "Chicken & Egg: Competition among Intermediation Service Providers," *RAND Journal of Economics*, 2003, *34* (2), 309–328.

Cain Miller, Claire, "Browser Wars Flare Again, on Little Screens," *The New York Times*, December 9, 2012.

Cohen, Adam, *The Perfect Store: Inside eBay*, Back Bay Books, 2003.

Crawford, Vincent P., "Lying for Strategic Advantage: Rational and Boundedly Rational Misrepresentation of Intentions," *American Economic Review*, 2003, *93* (1), 133–149.

_ **and Nagore Iriberri**, "Level-K Auctions: Can a Nonequilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private-Value Auctions?," *Econometrica*, 2007, *75* (6), 1721–1770.

_ **, Miguel A. Costa-Gomes, and Nagore Iriberri**, "Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications," *Journal of Economic Literature*, 2013, *51* (1), 5–62.

Edelman, Benjamin, "Does Google Leverage Market Power Through Tying and Bundling?," *Journal of Competition Law & Economics*, 2015, *11* (2), 365–400.

_ , "Uber Can't Be Fixed—It's Time for Regulators to Shut It Down," *Harvard Business Review Digital*, June 21, 2017.

_ **and Damien Geradin**, "Spontaneous Deregulation: How to Compete with Platforms That Ignore the Rules," *Harvard Business Review*, 2016, *94* (4), 80–87.

Farrell, Joseph and Paul Klemperer, "Coordination and lock-in: Competition with switching costs and network effects," *Handbook of industrial organization*, 2007, *3*, 1967–2072.

Fudenberg, Drew and Jean Tirole, "Pricing a network good to deter entry," *The Journal of Industrial Economics*, 2000, *48* (4), 373–390.

Gans, Joshua, David Hsu, and Scott Stern, "When Does Start-Up Innovation Spur the Gale of Creative Destruction?," *RAND Journal of Economics*, 2002, *33* (4), 571–586.

Gompers, Paul A. and Josh Lerner, "The Venture Capital Cycle," 1999.

Halaburda, Hanna and Yaron Yehezkel, "Focality advantage in platform competition," *Journal of Economics and Management Strategy*, 2018, *forthcoming.*

_ **, Bruno Jullien, and Yaron Yehezkel**, "Dynamic Competition with Network Externalities: How History Matters," *Working Paper*, 2018.

Ho, Teck-Hua, Colin Camerer, and Keith Weigel, "Iterated Dominance and Iterated Best Response in Experimental "p-Beauty Contests"," *American Economic Review*, 1998, *88* (4), 847–969.

Katz, Michael L. and Carl Shapiro, "Network externalities, competition, and compatibility," *The American Economic Review*, 1985, *75* (3), 424–440.

_ **and** _ , "Systems competition and network effects," *The Journal of Economic Perspectives*, 1994, *8* (2), 93–115.

Kets, Willemien and Alvaro Sandroni, "A Theory of Strategic Uncertainty and Cultural Diversity," *Working Paper*, 2017.

Khan, Lina M., "Amazon's Antitrust Paradox," *Yale Law Journal*, 2017, *126*, 710–805.

Klemperer, Paul, "Markets with consumer switching costs," *Quarterly Journal of Economics*, 1987, *102* (2), 375–394.

_ , "Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade," *Review of Economic Studies*, 1995, *62* (4), 515–539.

Kovacic, William E. and Carl Shapiro, "Antitrust Policy: A Century of Economic and Legal Thinking," *Journal of Economic Perspectives*, 2000, *14* (1), 43–60.

Nagel, Rosemarie, "Unraveling in Guessing Games: An Experimental Study," *American Economic Review*, 1995, *85* (5), 1313–1326.

Rivkin, Jan W. and Eric J. Van den Steen, "Microsoft's Search," *Harvard Business School Case 709-461*, 2009.

Rochet, Jean-Charles and Jean Tirole, "Platform competition in two-sided markets," *Journal of the European Economic Association*, 2003, *1* (4), 990–1029.

_ **and** _ , "Two-sided markets: a progress report," *RAND Journal of Economics*, 2006, *37* (3), 645–667.

Schumpeter, Joseph, *Capitalism, Socialism and Democracy*, Harper and Brothers, 1942.

Stahl, Dale O. and Paul W. Wilson, "Experimental evidence on players' models of other players," *Journal of Economic Behavior and Organizations*, 1994, *25* (3), 309–327.

Weyl, E Glen, "A price theory of multi-sided platforms," *American Economic Review*, 2010, *100* (4), 1642–72.

Yoffie, David B. and Michael A. Cusumano, *Competing on Internet time: lessons from Netscape and its battle with Microsoft*, Simon and Schuster, 1998.